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Drag Estimation

Based on the Aircraft Design Lecture Notes and Master Thesis of Roberto Segovia García. Hamburg University of Applied Sciences. 2017

In cruise we calculate the drag coefficient from

$$C_D = C_{D0} + \Delta C_{DW} + \frac{C_L^2}{\pi A e}$$

C_{D0} : Zero lift drag (Chapter 1)
ΔC_{DW} : Wave drag (Chapter 2)

For **cruise**

$$C_{D0} = C_{D0,clean} \quad ,$$

but for **take-off** (with initial climb) and **landing** (with approach) the zero lift drag coefficient has further components, because high-lift devices may be deployed and/or the landing gear may be extended. Especially the landing gear adds a considerable amount of drag.

$$C_{D0} = C_{D0,clean} + \Delta C_{D,flap} + \Delta C_{D,slat} + \Delta C_{D,gear}$$

For **cruise** the **Oswald Factor *e*** is calculated from the presentation "Estimating the Oswald Factor from Basic Aircraft Geometrical Parameters" (see under http://OPerA.ProfScholz.de "Method only"). Without any further calculation $e = e_{CR} = 0.85$ can be taken as a standard parameter (for moderate cruise Mach numbers).

For **take-off** (with initial climb) and **landing** (with approach) the **Oswald Factor** $e = e_{T/O} = 0.7$ can be taken as a standard parameter together with $e_{CR} = 0.85$. However, any other values of e_{CR} : $e_{T/O} = e_{CR} \cdot 0.7/0.85$.

1 Zero Lift Drag

$$C_{D0} = C_{D0,clean} + \Delta C_{D,flap} + \Delta C_{D,slat} + \Delta C_{D,gear}$$

"clean" means: neither flaps, slats, nor the landing gear are extended.

$$C_L = C_{L,max}\left(\frac{V_s}{V}\right)^2$$

V / *Vs* = 1.2 for take-off and initial climb
V / *Vs* = 1.3 for approach and landing

$$\Delta C_{D,flap} = 0.05\, C_L - 0.055$$
$$\text{for } C_L \geq 1.1 \quad .$$

Estimating C_{D0} "clean" from E_{max} :

$$\boxed{C_{D,0} = \frac{\pi \cdot A \cdot e}{4 \cdot E_{max}^2}}$$

(for E_{max} see Chapter 3)

Estimating C_{D0} "clean" from wetted area

$$C_{D,0} = C_{fe} \cdot \frac{S_{wet}}{S_W}$$

$$S_{wet} = S_{wet,F} + S_{wet,W} + S_{wet,H} + S_{wet,V} + n_E \cdot S_{wet,N} + n_E \cdot S_{wet,pylons}$$

See: *Aircraft Design Lecture Notes, Section 13 for wetted area calculation.*

Estimating C_{D0} in more detail from a drag built up:

$$C_{D0} = \sum_{c=1}^{n} C_{f,c} \cdot FF_c \cdot Q_c \cdot \frac{S_{wet,c}}{S_{ref}}$$

See: *Aircraft Design Lecture Notes, Section 13 for all required parameters.*

2 Wave Drag

2.1 Introduction

The aim of this Section is the analysis of the wave drag of the aircraft. The initial calculation of the wave drag in PrOPerA was based on the Boeing and Airbus philosophy (**Scholz 1999**), so the tool considers that the cruise Mach number was equal to drag divergence Mach number and the wave drag coefficient was a constant. For turbofans, this concept can be correct, but for the turboprops it is unknown if the aircraft flies at the drag divergence Mach number. For this reason, a method for the calculation of the wave drag is needed. With this implementation, the previous assumption is not needed and the wave drag is calculated for each design.

2.2 Calculation of the Drag Divergence Mach Number M_{DD}

2.2.1 Equation obtained from Shevell

Shevell 1980 describes an analytical method for the calculation of the crest critical Mach number, M_{CC}.

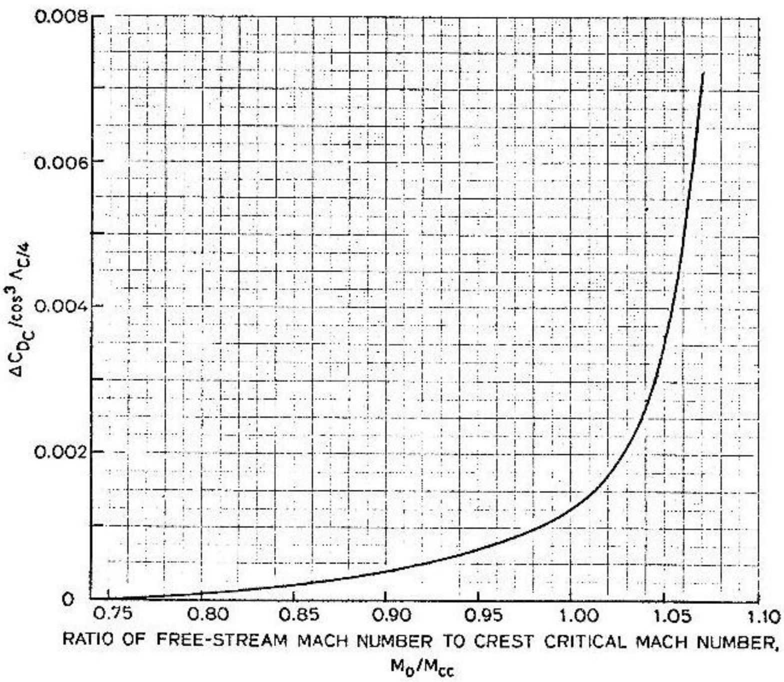

Figure 2.1 Wave drag curve obtained from **Shevell 1980**

From the Figure 2.1, the relation between M_{crit}, which is the critical Mach number, and M_{CC} is represented with

$$\frac{M_{crit}}{M_{CC}} = 0.74 \tag{2. 1}$$

The mathematical approximation for the Shevell's curve that has been realized is the Equation 2.2. The curve is represented in the Figure 2.2.

$$\frac{\Delta c_{D,wave}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \tag{2. 2}$$

Where $A = 0.00057$ and $B = 3.34821$.

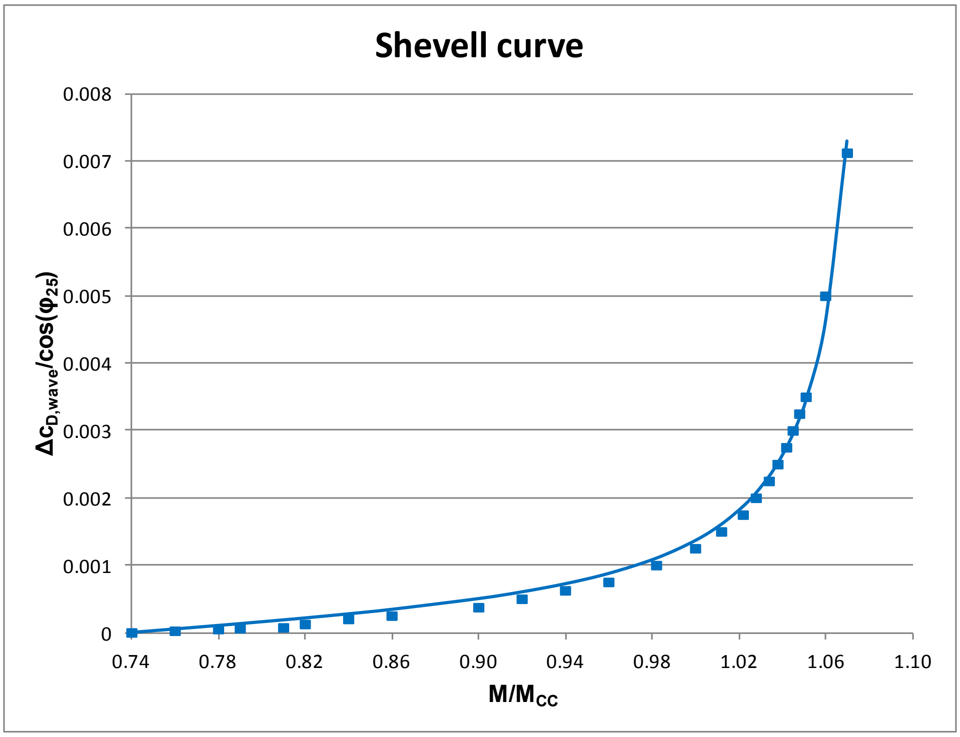

Figure 2.2 Representation of the approximation of the Shevell's curve

The definition that has been taken for the calculation of M_{DD} is given by Boeing and M_{DD} is defined as the Mach number for which the wave drag coefficient has an increment of 0.002, or 20 drag counts [CTS] (2.3). The Equation 2.4 represents the calculation of M_{DD}.

$$\Delta c_{D,wave} = 0.002 \tag{2. 3}$$

$$M_{DD} = \frac{(0.74 \cdot M_{CC})}{B} \cdot \left[\tan^{-1}\left(\frac{0.002}{\mathrm{A} \cdot \cos^3(\varphi_{25,w})}\right) + B\right] \tag{2. 4}$$

2.3 Calculation of the Wave Drag Curve

2.3.1 Analysis of the Real Wave Drag Curves

The calculation has been based on experimental data from some aircraft. The aircraft that have been studied are: A320-200, B727-200, B737-800, C-130H and BAe 146-200. The wave drag curves of B727-200, C-130H and BAe 146-200 have been obtained from **Roskam II 1985,** Figure 2.3.

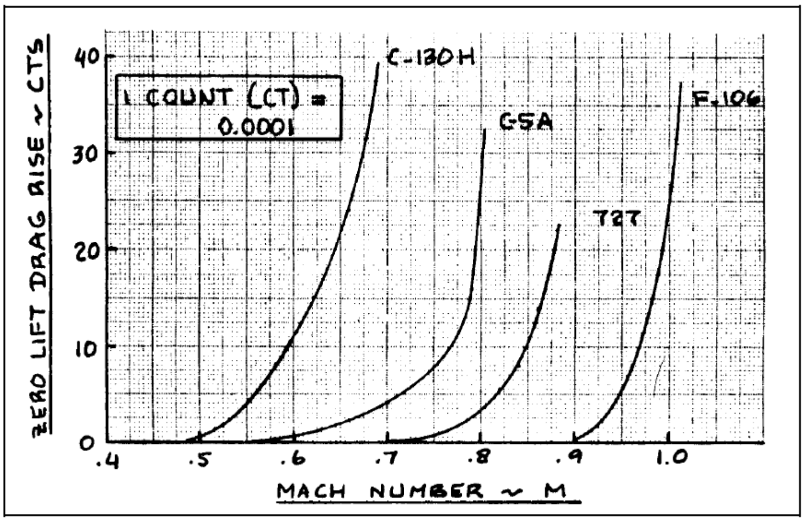

Figure 2.3 Wave drag curves of B727-200, C-130H and BAe 146-200, obtained from **Roskam II 1985**.

With this empirical information, M_{crit} and M_{DD} have been measured. The values of M_{DD} and M_{crit} for the five airplanes are represented in the Table 2.1.

Table 2.1 Values of M_{DD} and M_{crit} for the aircraft that have been analyzed

	A 320-200	B727-200	B737-800	C-130H	BAe 146-200
M_{crit}	0.60	0.70	0.60	0.49	0.53
M_{DD}	0.80	0.88	0.80	0.64	0.67

The wave drag curves have been approximated with equations that have the same structure as 2.5.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) + C\right) + D \tag{2. 5}$$

Due to the conditions of the problem, $\Delta c_{D,w}$ has to be zero at $M = M_{crit}$, so $C = -B$ and $D = 0$. The resultant equation is 2.6.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \tag{2. 6}$$

The values of the constants A and B are given in the Table 2.2 for each aircraft. The curves for the A320-200, B727-200, B737-800, C-130H and BAe 146-200 have been represented in the Figure 2.4. The representation is based on the Equation 2.6 with the constants of the Table 2.2.

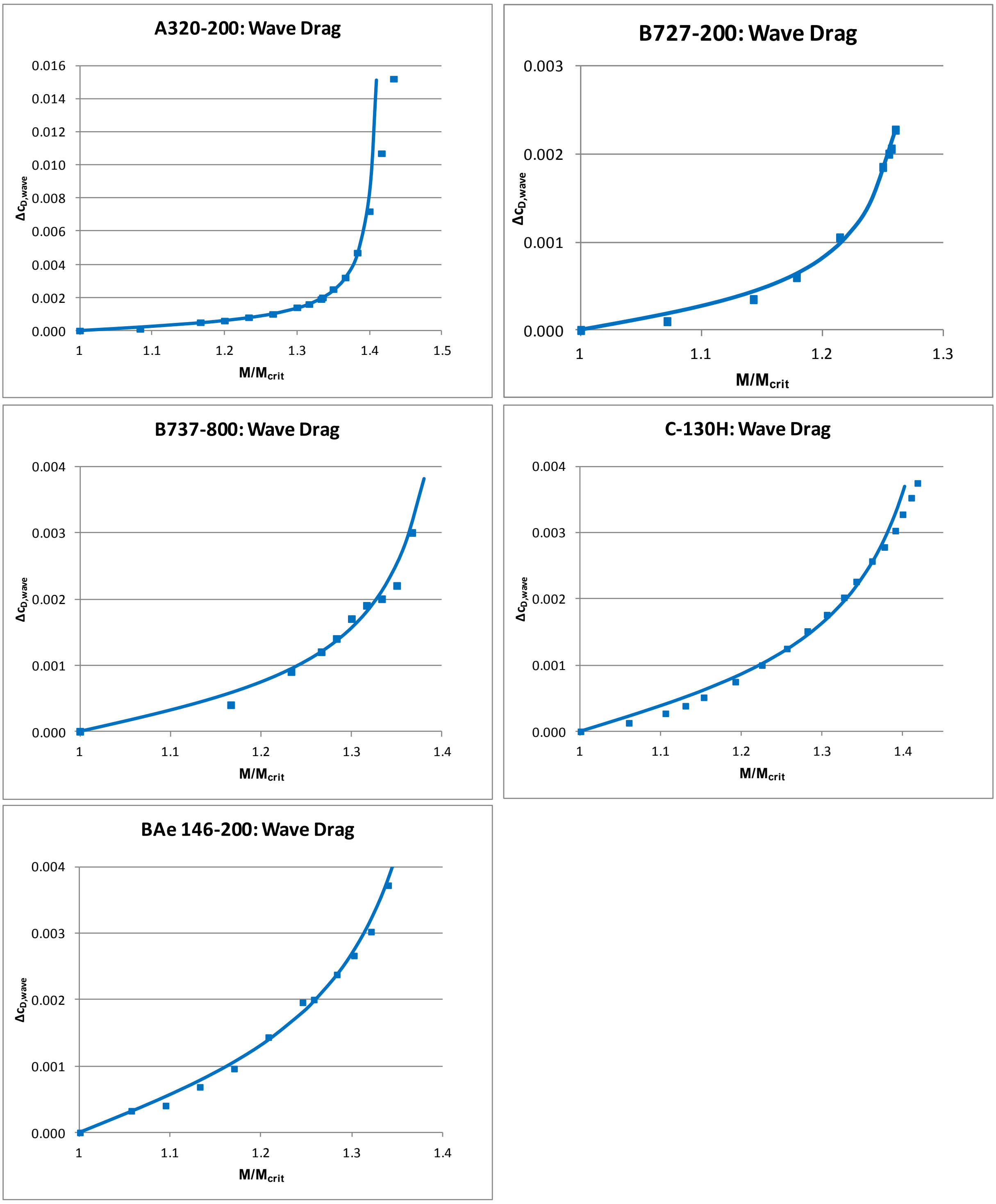

Figure 2.4 Wave drag curves for A320-200, B727-200, B737-800, C-130H and BAe 146-200

Table 2.2 Values of A and B for the approximation of the wave drag curves of the aircraft that have been analyzed

	A 320-200	B727-200	B737-800	C-130H	BAe 146
A	0.000885	0.000766	0.001171	0.001201	0.001765
B	3.734	5.257	3.543	3.126	3.457

2.3.2 Tangent Equation with the Effect of the Sweep Angle for the Calculation of the Wave Drag Curve

This equation has been developed based on the approximation of the Shevell's curve (2.2). The equation is represented by 2.7.

Limitation of the method:
M < M_crit (1 + pi/(2B))

$$\Delta c_{D,w} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \cdot \cos^3(\varphi_{25,w}) \quad (2.7)$$

The constants A and B are represented in the Table 2.3 and the SEE is represented in the Table 2.4, in [CTS]. The curve defined by 2.7 and the real curves (approximated with the Equation 2.6) are shown in the Figure 2.5.

Table 2.3 Values of A and B for only one "optimized" Tangent Equation for all aircraft listed in Table 2.2 represented by Equation 2.7

	Result
A	0.001272
B	3.477

Use these values for all aircraft not given in Table 2.2.

Table 2.4 SEE obtained for each aircraft, using the Tangent Equation based on Shevell's curve (**Shevell 1980**), Equation 2.7

	A 320-200	B727-200	B737-800	C-130H	BAe 146-200
SEE [CTS]	9.44	7.41	1.55	23.83	6.99

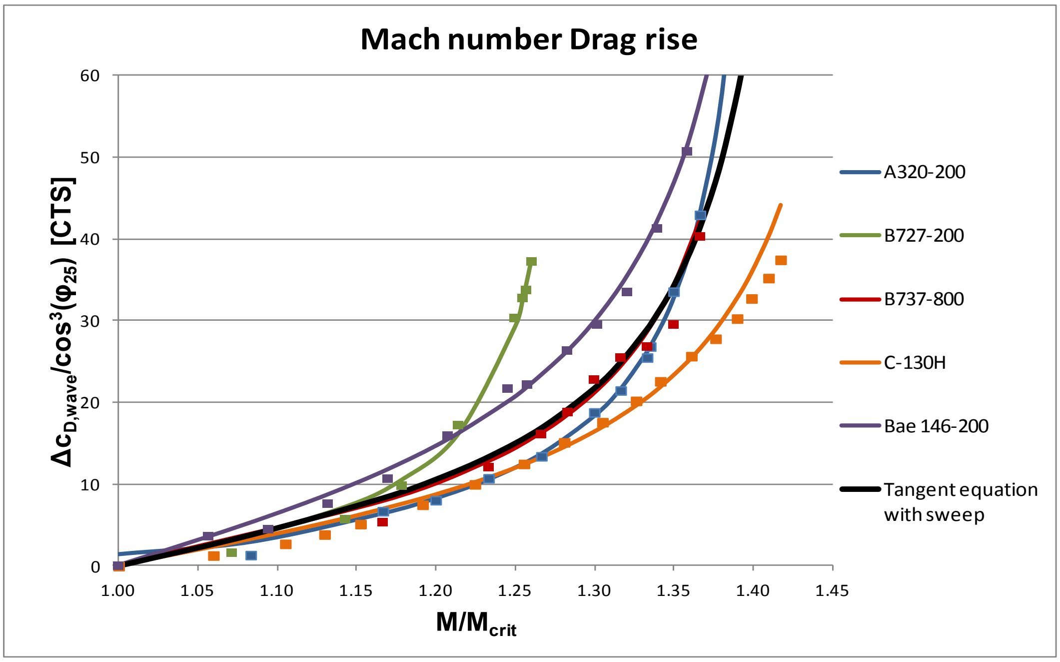

Figure 2.5 Representation of the Tangent Equation based on Shevell's curve (**Shevell 1980**), Equation 2.7, and the real curves (approximated with the Equation 2.6)

2.4 Calculation of the Critical Mach Number, M_{crit}

2.4.1 Calculation of M_{crit} obtained from the Tangent Equation with the Effect of Sweep Angle

The critical Mach number is obtained from the Equation 2.7. The resultant equation for the calculation of M_{crit} is 2.10.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \quad (2.8)$$

$$\frac{0.002}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M_{DD}}{M_{crit}}\right) - B\right) \quad (2.9)$$

Calculate M_crit for Equation 2.7 for all aircraft not in Table 2.2.

$$M_{crit} = \frac{B \cdot M_{DD}}{\tan^{-1}\left(\frac{0.002}{A \cdot \cos^3(\varphi_{25,w})}\right) + B} \quad (2.10)$$

The constants A and B are given in the Table 2.3.

3 Estimating Maximum Glide Ratio, E_{max}

$$E_{max} = k_E \sqrt{\frac{A}{S_{wet} / S_W}}$$

$S_{wet} / S_W = 6.0 \dots 6.2$ for typical passenger jets (see Fig. 1.1).

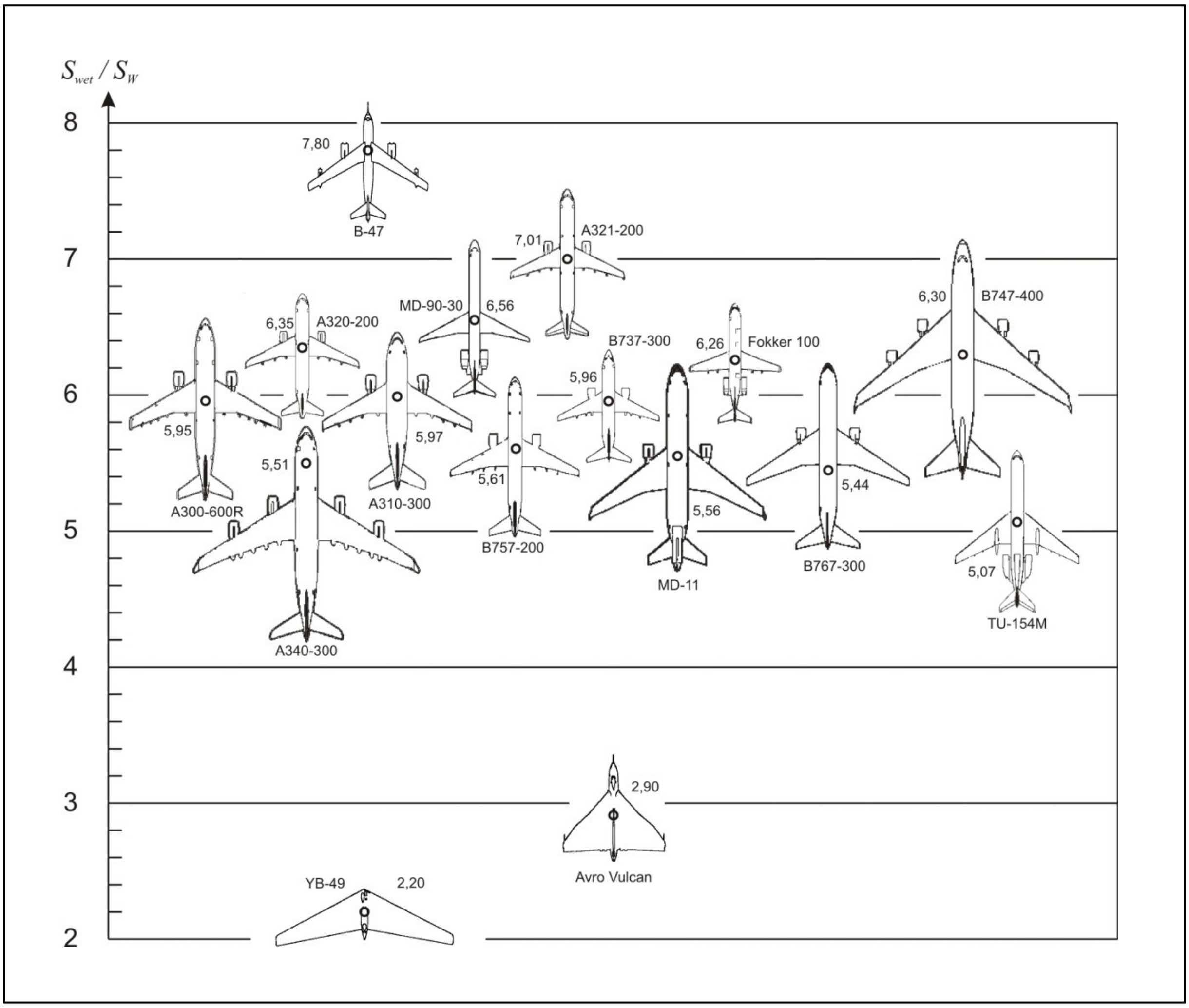

Fig. 1.1 Aircraft plan forms and their relative wetted area S_{wet} / S_W

Table 1.1 The equivalent skin-friction drag coefficient C_{fe} on the basis of general experience (**Roskam I**)

aircraft type	C_{fe} **- subsonic**
jets	0.003 ... 0.004
twins	0.004 ... 0.007
singles	0.005 ... 0.007

k_E calculated:

$$k_E = \frac{1}{2}\sqrt{\frac{\pi\ e}{\bar{c}_f}}$$

Oswald Factor e in cruise (see above).

k_E given as:

k_E	14.9 when calculated for standard parameters, which are considered to be: $e = 0.85$, $\bar{C}_f = C_{fe} = 0.003$.
$\bar{C}_f = C_{fe}$	see Table 1.1
k_E	15.8 according to data in Raymer's book

from our own statistics:

k_E	15.15 short range aircraft
k_E	16.19 medium range aircraft
k_E	17.25 long range aircraft

4 Calculating the Glide Ratio, *E*

For any lift coefficient (which may not be the optimum one):

$$E = \frac{2\ E_{max}}{\frac{1}{\left(\frac{C_L}{C_{L,md}}\right)} + \left(\frac{C_L}{C_{L,md}}\right)}$$

$$C_{L,md} = \frac{\pi\ A\ e}{2 E_{max}}$$

$$C_L / C_{L,md} = 1/\left(V / V_{md}\right)^2$$

Appendix:
Derivation of Equation for E_{max}

$$q = \frac{1}{2}\rho V^2$$

$$D = D_o + D_i$$

$$D_o = q\, C_{D_o} \cdot S_w = q\, \overline{C_f} \cdot S_{wet}$$

$$C_{D_o} \cdot S_w = \overline{C_f} \cdot S_{wet}$$

$$C_{D_o} = \overline{C_f} \cdot S_{wet}/S_w$$

$$C_D = 2\, C_{D_o} = 2\, C_{D_i}$$

at E_{max}

$$C_{D_o} = C_{D_i}$$

$$\overline{C_f} \cdot S_{wet}/S_w = \frac{C_L^2}{\pi A e}$$

$$C_L = \sqrt{\overline{C_f} \cdot S_{wet}/S_w \cdot \pi A e}$$

$$E_{max} = \frac{C_L}{C_D} = \frac{C_L}{2 C_{D_o}} = \frac{1}{2} \cdot \sqrt{\frac{\overline{C_f} \cdot S_{wet}/S_w \cdot \pi A e}{\overline{C_f}^2 \cdot (S_{wet}/S_w)^2}}$$

$$\boxed{E_{max} = \frac{1}{2}\sqrt{\frac{\pi A e}{\overline{C_f} \cdot S_{wet}/S_w}}}$$

$$E_{max} = \underbrace{\frac{1}{2}\sqrt{\frac{\pi e}{\overline{C_f}}}}_{K_E} \cdot \sqrt{\frac{A}{S_{wet}/S_w}}$$

$$\boxed{E_{max} = K_E \cdot \sqrt{\frac{A}{S_{wet}/S_w}}}$$