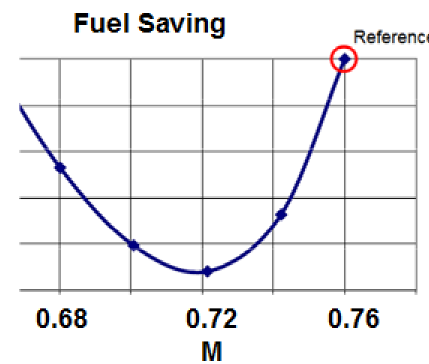
Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

AIRCRAFT DESIGN AND SYSTEMS GROUP (AERO)

Flying Low and Slow

(and the Tools for its Calculation)



Dieter Scholz

Hamburg University of Applied Sciences

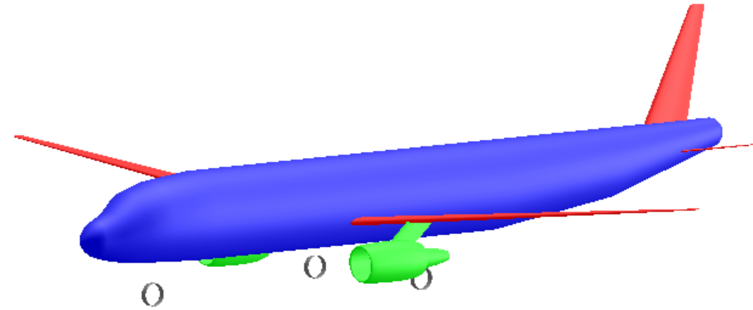
12th European Workshop on Aircraft Design Education (EWADE) 2015

Delft, 10. September 2015

Content

Flying Low and Slow with New Aircraft Designs

- **Standard Jet Configuration**
  "The Rebel"
  Optimization for minimum fuel



- **Standard Prop Configuration**
  "Smart Turboprop"
  Optimization for minimum DOC

Genetic algorithm (Differential Evolution) proposes parameters. Aircraft „designed“ automatically in EXCEL. About 2000 feasible designs tested in one optimization run.

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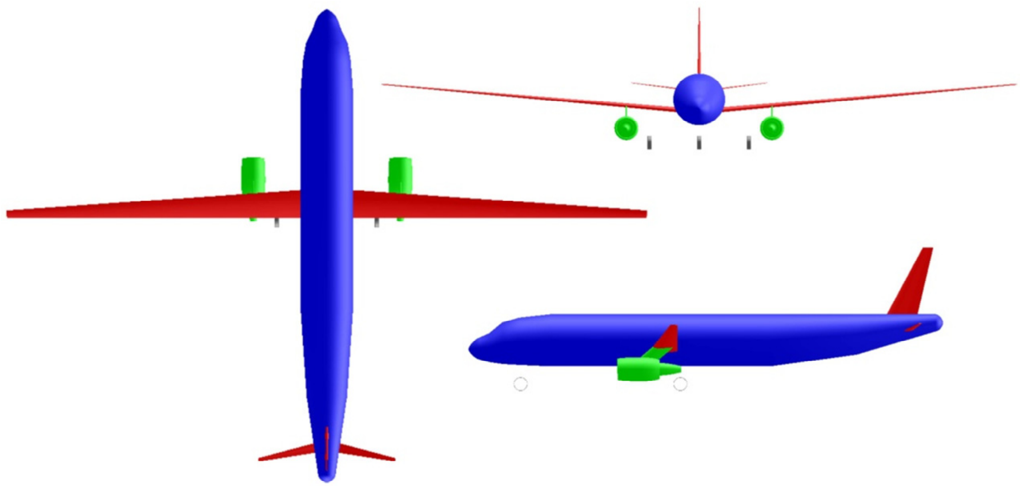
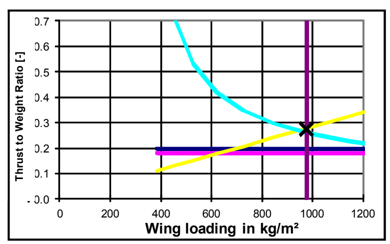
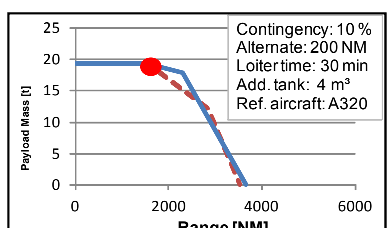
Standard Jet Configuration: "The Rebel"



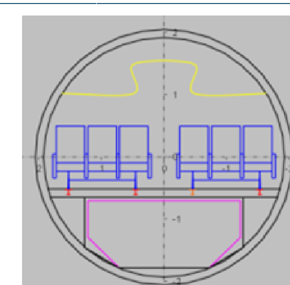
Early conceptual design

Parameter	Value	Deviation from A320*
Requirements		
m_{MPL}	19256 kg	0 %
R_{MPL}	1510 NM	0 %
M_{CR}	0.55	- 28 %
max(s_{TOFL} , s_{LFL})	2700 m	+ 53 %
n_{PAX} (1-cl HD)	180	0 %
m_{PAX}	93 kg	0 %
SP	28 in	- 3 %

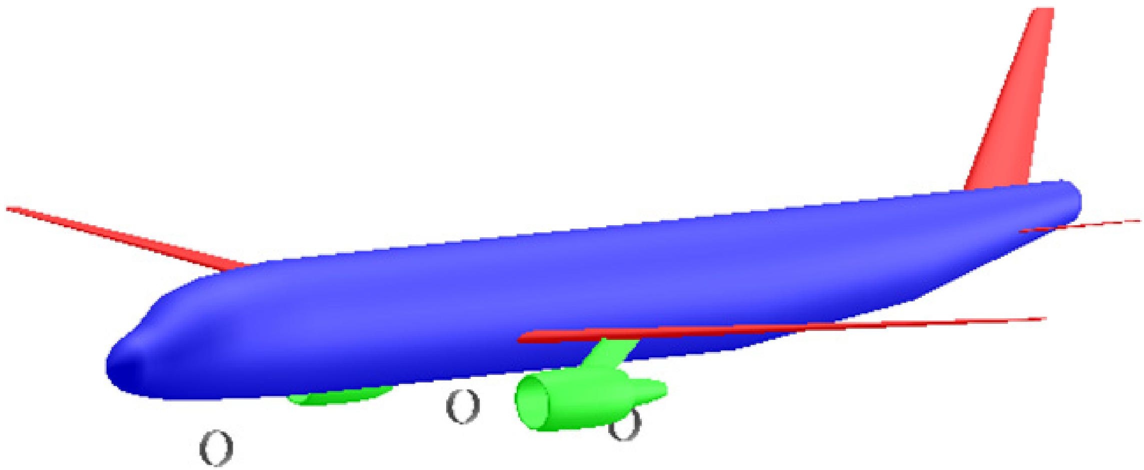




Parameter	Value	Deviation from A320*
Main aircraft parameters		
m_{MTO}	66000 kg	- 10 %
m_{OE}	39200 kg	- 5 %
m_F	7500 kg	- 42 %
S_W	68 m²	- 45 %
$b_{W,geo}$	48.5 m	+ 42 %
$A_{W,eff}$	34.8	+ 266 %
E_{max}	26.1	+ 48 %
T_{TO}	89100 N	- 20 %
BPR	15.5	+ 158 %
SFC	1.03E-5 kg/N/s	- 37 %
h_{ICA}	30000 ft	- 23 %
s_{TOFL}	2490 m	+ 41 %
s_{LFL}	2110 m	+ 45 %
t_{TA}	32 min	0 %

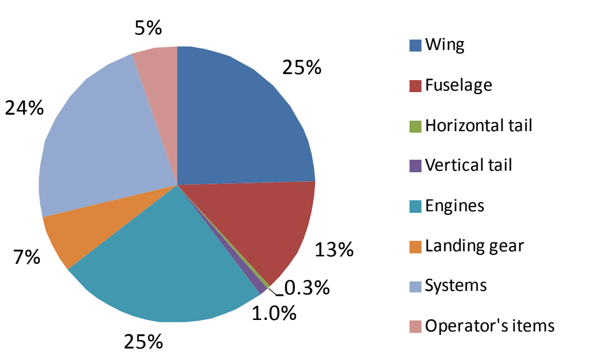


Standard Jet Configuration: "The Rebel"

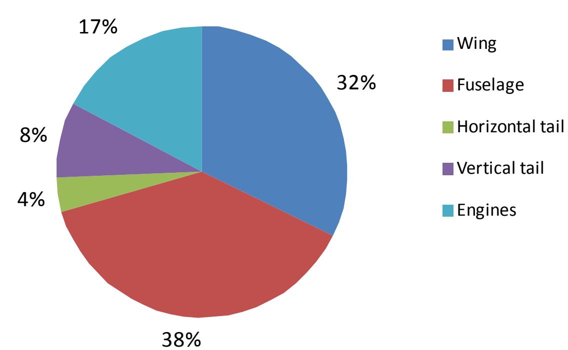


Parameter	Value	Deviation from A320*
DOC mission requirements		
R_{DOC}	750 NM	0 %
$m_{PL,DOC}$	19256 kg	0 %
EIS	2030	-----
c_{fuel}	1.44 USD/kg	0 %
Results		
$m_{F,trip}$	3700	- 36 %
$U_{a,f}$	3070	+ 6 %
DOC (AEA)	93 %	- 7 %

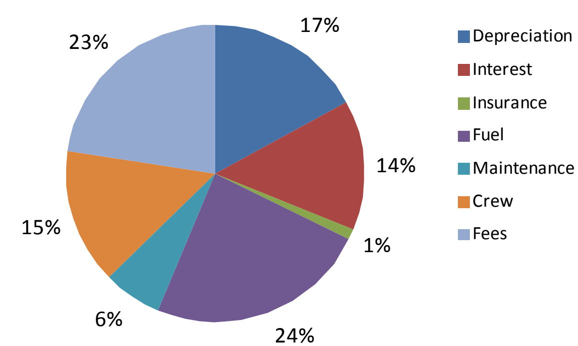
Operating empty mass breakdown



Component drag breakdown

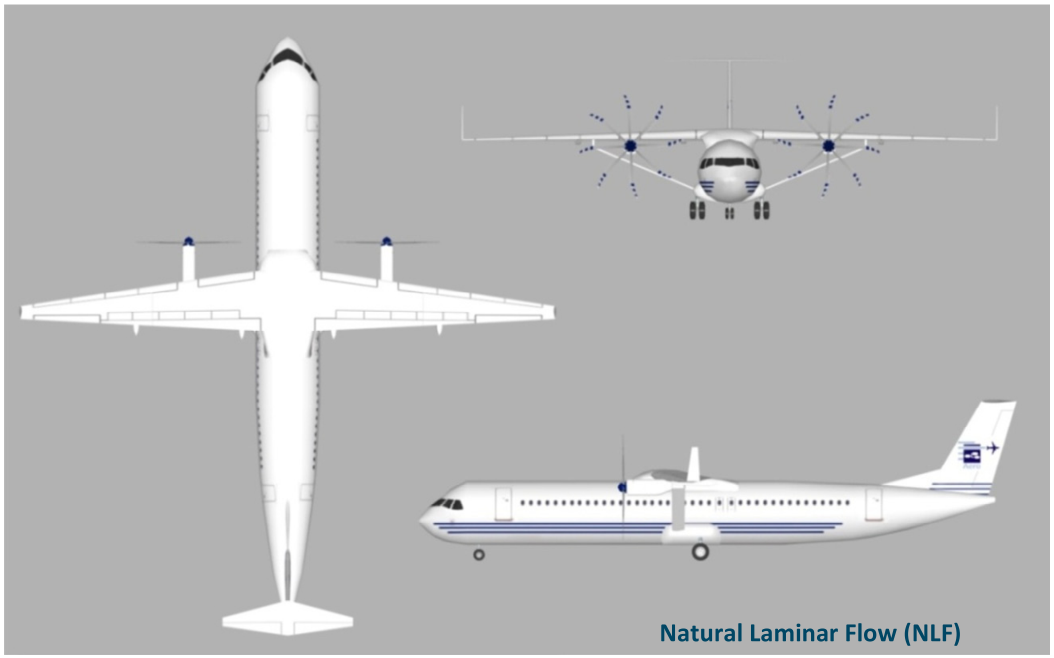


Direct operating cost breakdown

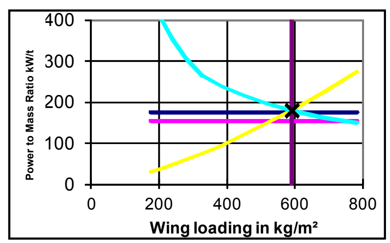


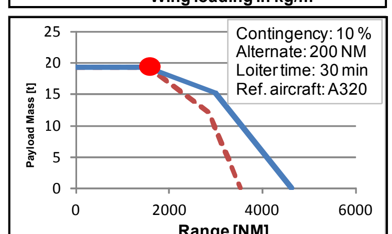
Flying Low and Slow with New Aircraft Designs

Standard Prop Configuration: "Smart Turboprop"

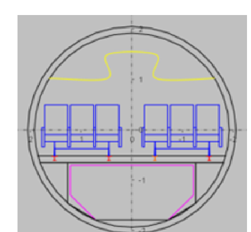


Parameter	Value	Deviation from A320*
Requirements		
m_{MPL}	19256 kg	0 %
R_{MPL}	1510 NM	0 %
M_{CR}	0.51	- 33 %
max(s_{TOFL} , s_{LFL})	1770 m	0 %
n_{PAX} (1-cl HD)	180	0 %
m_{PAX}	93 kg	0 %
SP	29 in	0 %





Parameter	Value	Deviation from A320*
Main aircraft parameters		
m_{MTO}	56000 kg	- 24 %
m_{OE}	28400 kg	- 31 %
m_F	8400 kg	- 36 %
S_W	95 m²	- 23 %
$b_{W,geo}$	36.0 m	+ 6 %
$A_{W,eff}$	14.9	+ 57 %
E_{max}	18.8	≈ + 7 %
$P_{eq,ssl}$	5000 kW	------
d_{prop}	7.0 m	------
η_{prop}	89 %	------
$PSFC$	5.86E-8 kg/W/s	------
h_{ICA}	23000 ft	- 40 %
s_{TOFL}	1770 m	0 %
s_{LFL}	1300 m	- 10 %
t_{TA}	32 min	0 %



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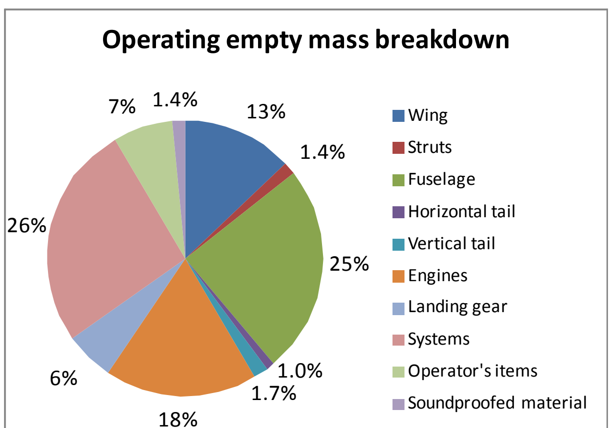
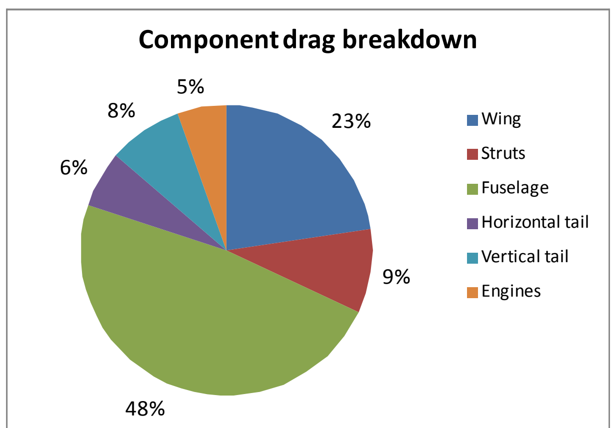
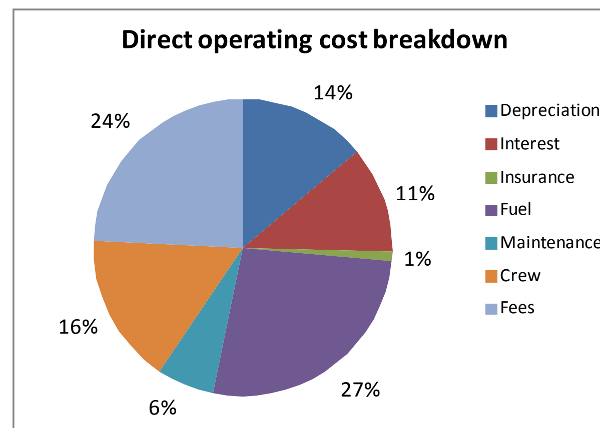
Standard Prop Configuration: "Smart Turboprop"

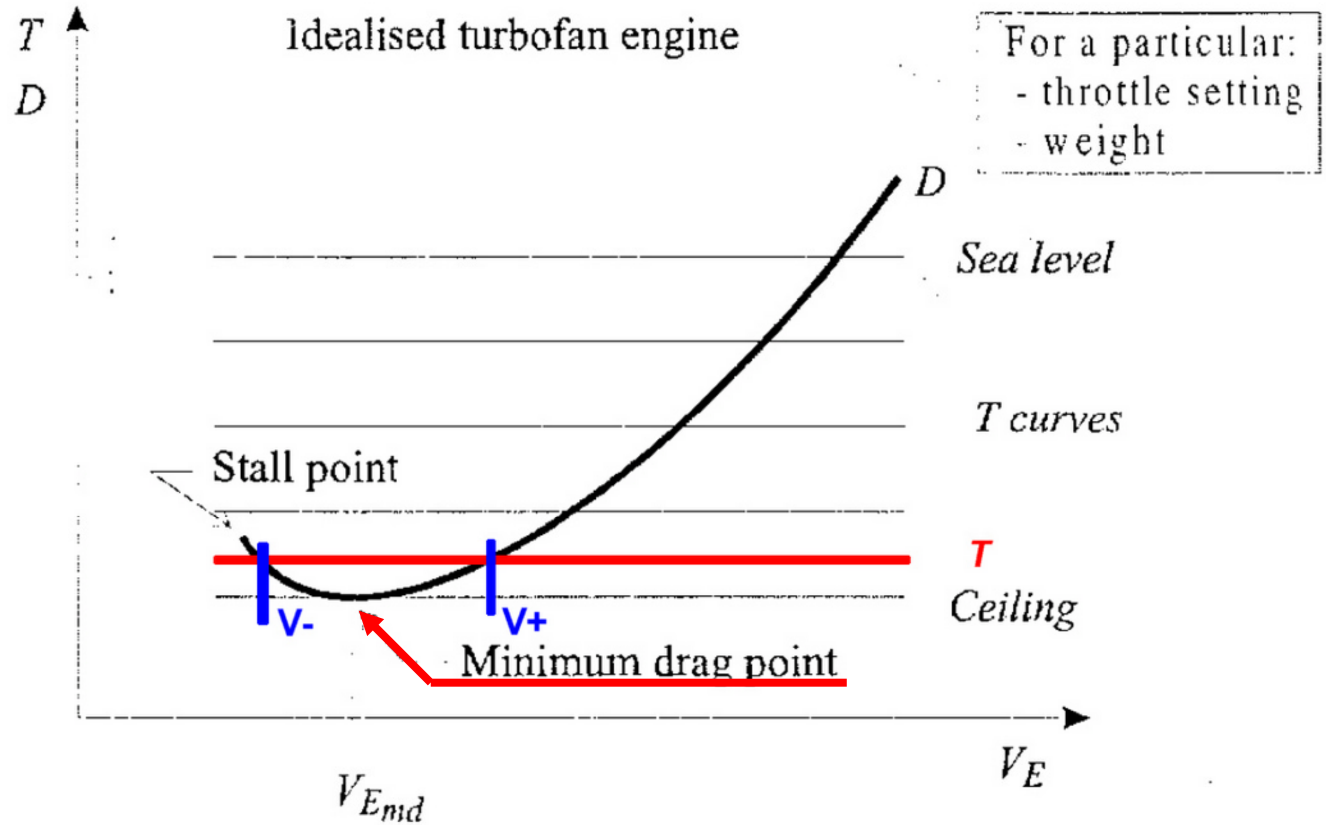
Parameter	Value	Deviation from A320*
DOC mission requirements		
R_{DOC}	755 NM	0 %
$m_{PL,DOC}$	19256 kg	0 %
EIS	2030	-----
c_{fuel}	1.44 USD/kg	0 %
Results		
$m_{F,trip}$	3700 kg	- 36 %
$U_{a,f}$	3600 h	+ 5 %
DOC (AEA)	83 %	- 17 %







A/C Performance



Often claimed:

"There is one speed for minimum drag!" Really only one?

flying lower: $mg = L = \frac{1}{2}\rho V^2 C_{L,md} S_W$

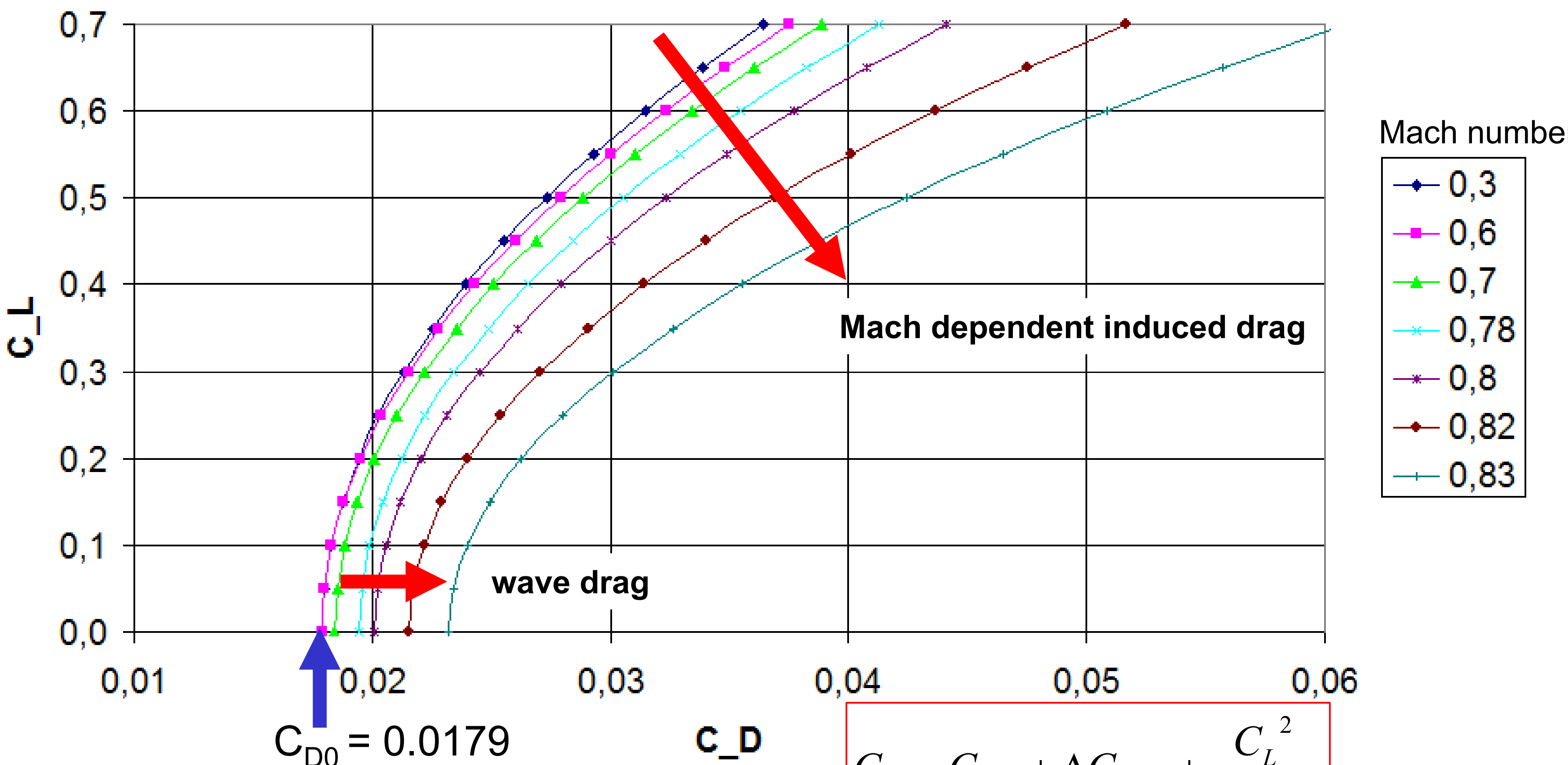
The Pilot's View of Flying Low and hence Fast

Lower altitude => Higher Speed of Sound => **Higher True Airspeed** (**cruise Mach number remains constant**

High Speed & high density => very low lift coefficient => very low L/D

=> Extremely high fuel consumption !

Drag Polar (Airbus A320, approximated, based on the following equations)



$$C_D = C_{D,0} + \Delta C_{D,W} + \frac{C_L^{\ 2}}{\pi \ A \ e}$$

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Induced Drag Prediction Method (Nita 2012)

$$C_{D,i} = \frac{C_L^{\,2}}{\pi A e} \qquad e = \frac{k_{e,M}}{Q + P \pi A} \qquad Q = \frac{1}{e_{theo} \cdot k_{e,F}} \qquad P = K C_{D,0} \qquad K = 0{,}38$$

from one of many handbook methods

Fuselage: $k_{e,F} = 1 - 2\left(\frac{d_F}{b}\right)^2$

Mach: $k_{e,M} = a_e \left(\frac{M}{M_{comp}} - 1\right)^{b_e} + c_e$

$a_e < 0; \quad c_e = 1$

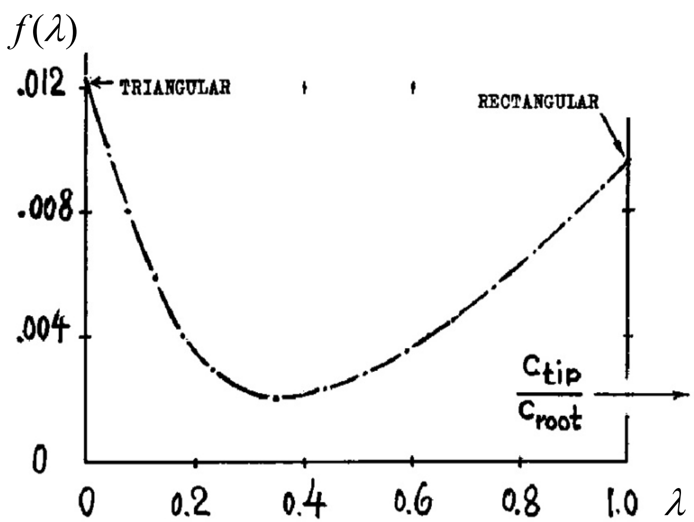
Generic parameters:

$a_e = -0.00152$

$b_e = 10.82$

$c_e = 1$

$M_{comp} = 0.3$



Hörner 1965

for unswept wings:

$$e_{theo} = \frac{1}{1 + f(\lambda) \cdot A}$$

Drag Polar

NACA Report 921

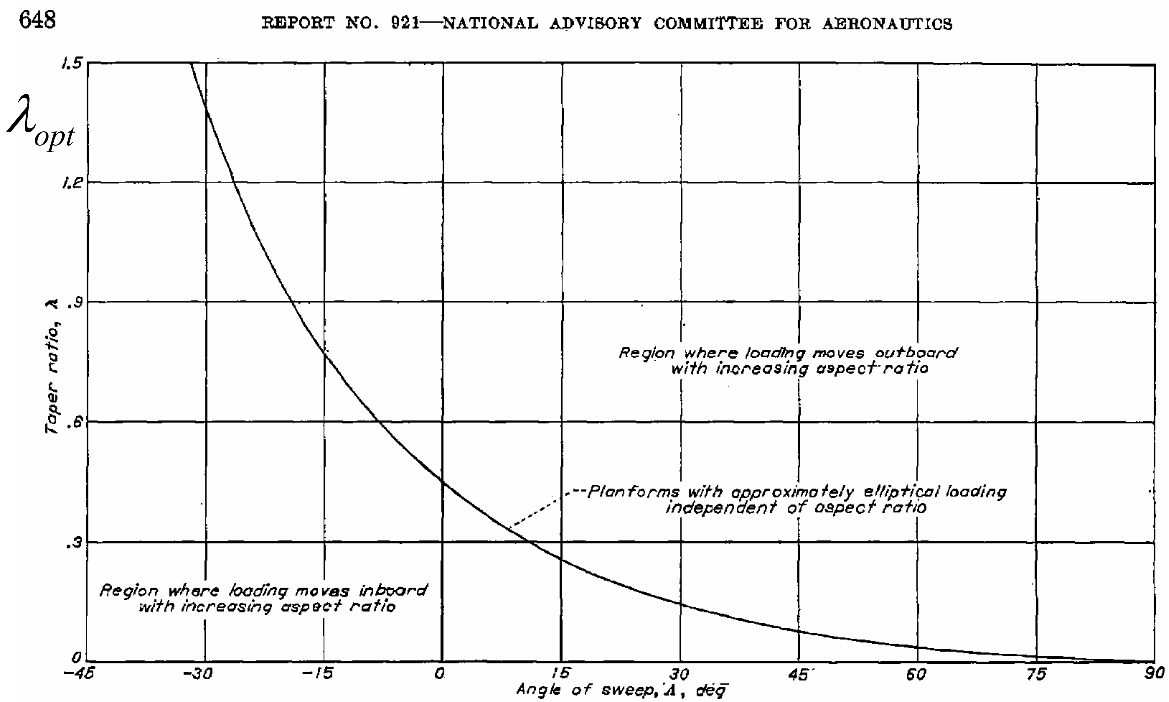


FIGURE 21.—Relation of taper ratio to sweep angle required for approximately elliptical loading.

φ_{25}

for all sweep angles φ_{25}:

$$\lambda_{opt} = 0.45 \cdot e^{-0.0375 \cdot \varphi_{25}}$$

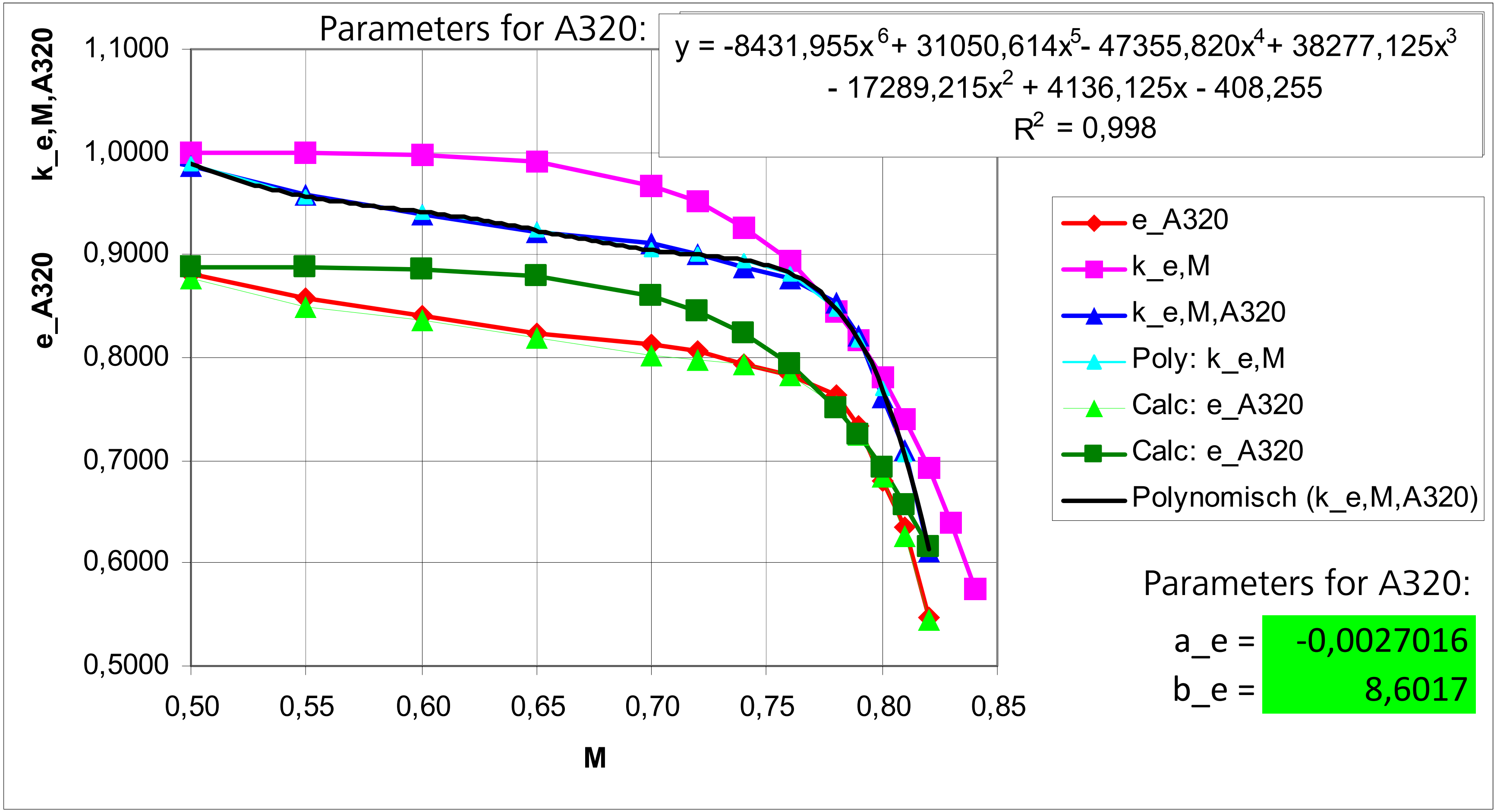
$$\Delta\lambda = -0.357 + 0.45 \cdot e^{-0.0375 \cdot \varphi_{25}}$$

φ_{25} in deg

$$e_{theo} = \frac{1}{1 + f(\lambda - \Delta\lambda) \cdot A}$$

$$f(\lambda - \Delta\lambda) = 0.0524\,(\lambda - \Delta\lambda)^4 - 0.15(\lambda - \Delta\lambda)^3 + 0.1659(\lambda - \Delta\lambda)^2 - 0.0706(\lambda - \Delta\lambda) + 0.0119$$

Mach Dependent Induced Drag (A320)



Wave Drag Prediction Method

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right)$$

Shevell 1980:
A = 0.00057, B = 3.348

Own proposal
of generic parameters (from 5 A/C):
A = 0.00127, B = 3.4766

	A 320-200	B727-200	B737-800	C-130H	BAe 146-200
M_{crit}	0.60	0.70	0.60	0.49	0.53
M_{DD}	0.80	0.88	0.80	0.64	0.67

	A 320-200	B727-200
A	0.001	0.001
B	3.734	5.257

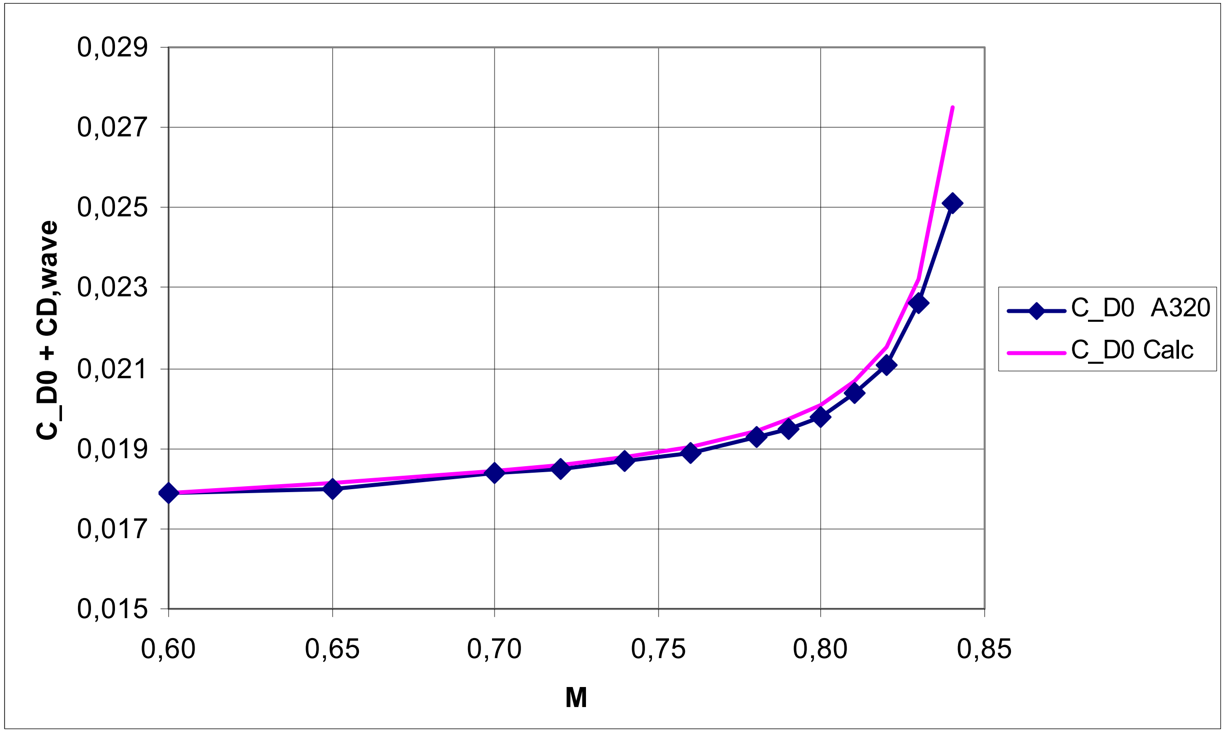
B737-800	C-130H	BAe 146
0.001	0.001	0.002
3.543	3.126	3.457

In case Mcrit is not given:

$$M_{crit} = \frac{M_{DD}}{1+\left(\frac{0.002}{a \cdot \cos^3(\varphi_{25,w})}\right)^{1/b}}$$

with a = A, b = B

A320:
C_{D0} = 0.0179



The SFC Paradox

$c_T = c$: **Thrust-Specific Fuel Consumption**

$c_T = \dot{m}_F / T$

$c_T = 16 \cdot 10^{-6}$ kg/N/s (typical for **jet**)

$$H = \frac{E}{m}$$ Heating value. Kerosene: H = 42.5 MJ/kg

$$\dot{E} = P = \dot{m}_F \, H$$

$$\dot{m}_F \cdot H \cdot \eta = P = T \cdot V = D \cdot V$$

η : overall efficiency of the flight

$$c_T \cdot T \cdot H \cdot \eta \overset{!}{=} T \cdot V$$

$$c_T \cdot H \cdot \eta = V$$

$$\eta = \frac{V}{c_T \cdot H}$$

Efficiency increases to any value if only speed is increased.
=> **Paradox** !

$\eta = 1$:
V = 680 m/s, M = 2 (MSL)

Deriving a Basic SFC as a Function of Speed or Mach Number

$c_P = c'$: Power-Specific Fuel Consumption

$c_P = \dot{m}_F / P / t$

$c_P = 0.075 \cdot 10^{-6}$ kg/W/s (typical for turboprop)

$$\dot{m}_F = c\,T = c'P = c'TV$$

$$c = c'V$$

c' is slope …

$$M = V/a \qquad V = M \cdot a$$

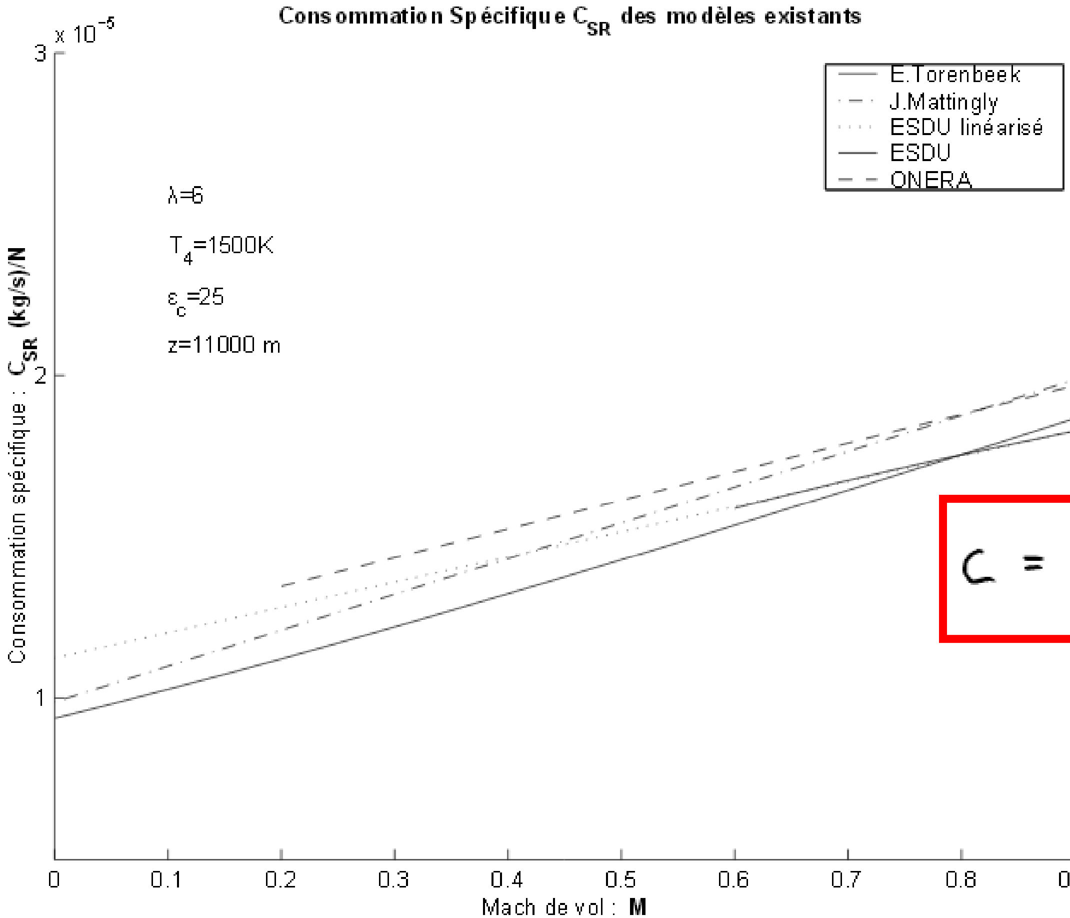
$$a = a_0 \cdot \sqrt{\frac{T}{T_0}} = a_0 \cdot \sqrt{\theta}$$

$$c = c' \cdot M \cdot a_0 \cdot \sqrt{\theta}$$

$$c = \underbrace{c' \cdot a_0 \cdot \sqrt{\theta}}_{a^*} \cdot M$$

a* is slope …

The Basic SFC Function



… a* is slope

$$c = c_{corr} \cdot (c_1 + c_2 \cdot M) \sqrt{\frac{T}{T_0}}$$

c_1	1,13E-05
c_2	1,25E-05
c_corr	0,92

Élodie Roux 2002

Flying Low and Slow with a Given Aircraft

Preparation

$$B_s = \frac{V \cdot E}{c \cdot g}$$ Breguet - Faktor

$$\frac{m_F}{m_{TO}} = 1 - M_{ff} = 1 - e^{-R/B_s}$$ relative fuel consumption

$$E_{max} = \sqrt{\frac{\pi A e}{C_{D0} + \Delta C_{D0,W}}}$$ $E = L/D$

$$\frac{E}{E_{ref}} = \sqrt{\frac{k_{e,M}}{k_{e,M,ref}} \cdot \frac{C_{D0} + \Delta C_{D0,W,ref}}{C_{D0} + \Delta C_{D0,W}}}$$

if the aircraft is unchanged and C_L is kept constant E only depends on $k_{e,M}$ and $\Delta C_{D0,W}$

The Reference

$M_{ref}, \quad h_{ref}, \quad C_{L,ref}, \quad E_{ref}$

$h_{ref} \rightarrow a_{ref}, \quad T_{ref}, \quad \rho_{ref}$

$M_{ref},\ a_{ref} \rightarrow V_{ref}$

Breguet $\Rightarrow C_L = const \Rightarrow E = const$

The New Flight Condition

$C_L = const$

$V \rightarrow \rho \rightarrow h \rightarrow T \rightarrow a \rightarrow M \rightarrow E$

$M \rightarrow c$

$E,\ c \rightarrow$

$V \rightarrow B_s \rightarrow M_{ff} \rightarrow m_F / m_{TO}$

More Basics

$$mg = \frac{1}{2} \rho_{ref} \cdot V_{ref}^2 \cdot C_L \cdot S_W$$

$$= \frac{1}{2} \rho \cdot V^2 \cdot C_L \cdot S_W$$

$$\rho = \rho_{ref} \cdot \frac{V_{ref}^2}{V^2}$$

$$T = T_0 - L \cdot h$$

$$a = a_0 \sqrt{\frac{T}{T_0}}$$

$$M = V/a$$

Troposphäre:

$$\rho/\rho_0 = (1 - K_a \cdot h)^{K_{exp}}$$

$$h = \frac{1}{K_a}\left[1 - \left(\frac{\rho}{\rho_0}\right)^{1/K_{exp}}\right]$$

$K_a = 2.2558 \cdot 10^{-5}$ 1/m

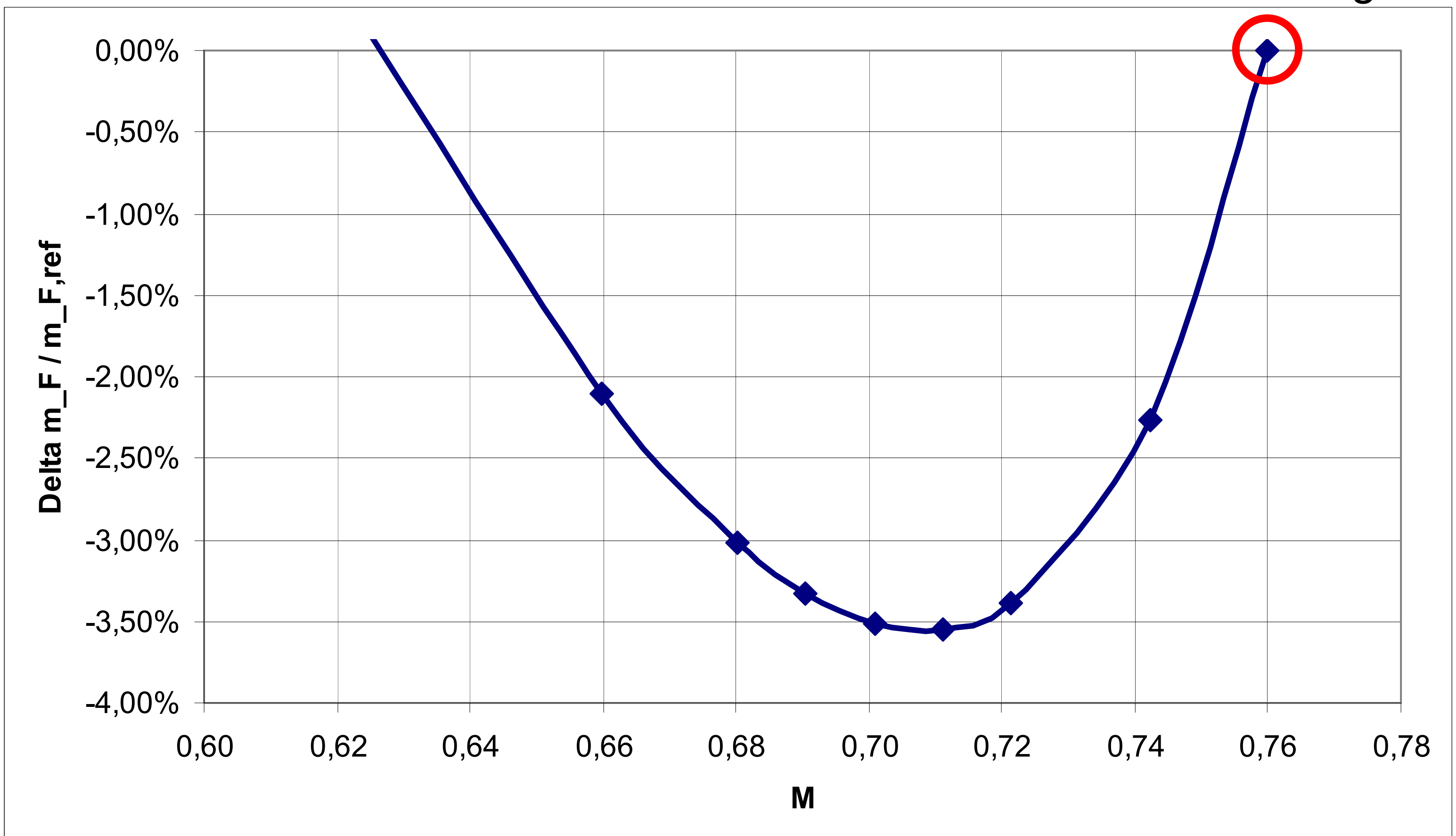
$K_{exp} = 4.25588$

Numbers (A320)

R	1510 NM	ae	-0,00152
R	2796520 m	be	10,82
		k_e_Mref	0,8450
Eref	17,9		
Mref	0,76	exp	4,25588
C_Lref	0,7	ka	2,26E-05 1/m
href	11000 m	rho0	1,225 kg/m³
aref	295	T0	288,15 K
Tref	216,65 K	L	0,0065 K/m
rhoref	0,36392 kg/m³	a0	340,294 m/s
c_1	1,13E-05		
c_2	1,25E-05		
c_corr	0,92		
g	9,81 m/s		
Vref	224 m/s		

Results

Reference:
long range cruise



Results

V	m/s	224	220	215	210	205	200	190
rho	kg/m³	0,364	0,378	0,396	0,415	0,435	0,458	0,507
h	m	10999	10698	10333	9955	9563	9158	8300
T	K	217	219	221	223	226	229	234
a	m/s	295	296	298	300	301	303	307
M		0,76	0,74	0,72	0,70	0,68	0,66	0,62
k_e_M		0,8450	0,8988	0,9399	0,9651	0,9802	0,9891	0,9970
CD0W		0,0011	0,0009	0,0007	0,0005	0,0004	0,0003	0,0001
E		17,9	18,6	19,1	19,4	19,7	19,8	20,0
c	kg/N/s	1,66E-05	1,65E-05	1,64E-05	1,63E-05	1,61E-05	1,60E-05	1,58E-05
Bs	m	2,47E+07	2,53E+07	2,56E+07	2,56E+07	2,55E+07	2,52E+07	2,45E+07
Mff		0,893	0,895	0,896	0,897	0,896	0,895	0,892
m_F/m_TO		0,1072	0,1048	0,1036	0,1034	0,1040	0,1049	0,1077
fuel saving		0,00%	-2,26%	-3,39%	-3,51%	-3,01%	-2,11%	0,48%

- *E* = ***L/D* increases** continuously with flying slower (down to *M* = 0.3).
- Thrust-specific fuel consumption *c* = **SFC decreases** with flying slower.
- The **Breguet factor** B_s is **proportion to speed** and decreases once *E* stops increasing with substancial rate.
- Fuel consumption decreases as long as the Breguet factor B_s increases.

Summary and Conclusions

- **Flying slower** gets you on a better drag polar
  (this is true also below the critical Mach number)

- The best lift coefficient has to be maintained
- This can be done by letting the design find its optimum condition with respect to
  altitude and wing area and …

- Given aircraft have to accept the given wing area and can fly lower when slower

- **An example calculation showed fuel burn reduction of 3.5 %
  at 0.05 Mach less than reference Mach number (0.76)**

- **This could be done today(!) with all aircraft(!) and would also reduce contrails**

- **But: Productivity goes down and DOC go (most probably) up!
  This is "only" a financial question, however decisive!**