Hochschule für Angewandte Wissenschaften Hamburg

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AIRCRAFT DESIGN AND SYSTEMS GROUP (AERO)

The Cubic Wing Loading Parameter in Passenger Aircraft Preliminary Sizing

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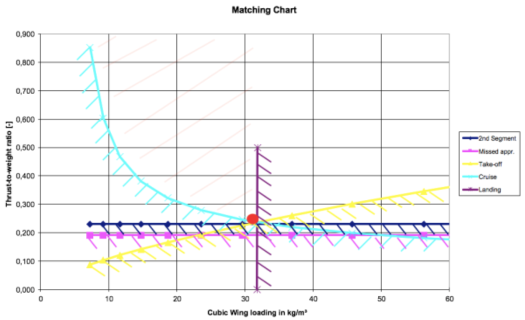

$$CWL = \frac{m_{MTO}}{S^{3/2}}$$

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Abstract

Purpose – This thesis investigates the parameter Cubic Wing Loading (CWL). It is the mass of the aircraft divided by the wing surface area taken to the power of 3/2. As such the unit of the denominator is converted from m² to m³ and CWL has the unit of kg/m³. Classical Wing Loading (WL) is aircraft mass divided by wing area. It is investigated, if CWL (unlike WL) is independent of aircraft size, if it has advantages in preliminary aircraft design, and if it can be used as a basis for interesting correlations.

Methodology – Aircraft preliminary sizing equations for passenger jet aircraft are rewritten to replace WL with CWL. Aircraft statistical data are investigated with respect to CWL.

Findings – Like WL, also CWL depends on aircraft size. CWL of different aircraft vary strongly. CWL introduced to preliminary sizing leads to additional (but manageable) iterations compared to preliminary sizing based on WL. Correlations with other aircraft design parameters are weak and no relation with accident rates for high CWL aircraft is found. However, a new performance factor related with the speed range of different aircraft can be set considering the product of CWL and T/W. Further, larger aircraft with low CWL built against square-cube law are proved to lead to larger planes with better fuel economy and power to weight ratio, heading to a maximum reduction of m_{OE}/m_{MTO}, m_F/ m_{MTO} and T/W of 0.005, 0.01 and 0.0025 respectively.

Research Limitations – 209 airplanes are studied for statistical correlations. The study is reduced to 72 models in some sections due to the hitch to find more specific data for other planes.

Practical Implications – There are no advantages to replace WL by CWL in passenger aircraft preliminary sizing.

Originality – This seems to be the first report to fully investigate CWL with respect to passenger aircraft and to offer a related user-friendly preliminary sizing spreadsheet.

Motivation

Air traffic is expected to continue to growing in the next decades. New planes, more efficient, faster, and safer will be required to be built by manufacturers. Therefore, aircraft design is and will continue to be an interesting area of research and development. Nowadays, aircraft design theories are based in two basic parameters, the ratio between thrust of the engines and weight of the plane (Thrust to Weight Ratio), and the one between mass and wing area (Wing Loading). Based on these two parameter the design of an aircraft can be determined when taking into account also the requirements.

Radio Controlled (RC) plane designers are used to work with another parameter, the ratio between the mass of the aircraft and the wing surface to the power of 1.5, known as Cubic Wing Loading. This parameter treated as a parameter independent of size for RC planes and it permits to compare models in different scales and it is directly related to their flight characteristics and their performance. Thus, it gives important information for designers and users.

Authors have considered recently the possibility to introduce this parameter into **passenger aircraft design**. However, it has not been investigated comprehensively, which advantages can this parameter may bring to passenger aircraft design.

Table of Content

Introduction

Objectives / Research Questions

- Is cubic wing loading rather a constant value for passenger aircraft of comparable design?
- Is it possible to rewrite aircraft design theory based in this new parameter?
- What are the benefits or disadvantages of applying cubic wing loading into aircraft design?
- What is the current knowledge of this parameter and what can be investigate further?

State of the Art

Dimensional Analysis

- **WL:**

$$\frac{m}{S} \sim \frac{l^3}{l^2} \sim l$$

[kg/m²]

- **CWL:**

$$\frac{m}{S^{\frac{3}{2}}} \sim \frac{l^3}{l^{2\,\frac{3}{2}}} \sim \frac{l^3}{l^3} \sim 1$$

[kg/m³]

Flyability Levels

model	mass	wing surface	WL	CWL
small model	0.6	0.15	4	10.33
basic model	1.7	0.3	5.7	10.35
big model	5.4	0.65	8.3	10.30

level	CWL range	typical type(s)
1	0.00-2.99	indoor
2	3.00-4.99	backyard
3	5.00-6.99	park Flyers
4	7.00-9.99	sport planes & Trainers
5	10.00-13.99	advanced sport
6	14.00-16.99	expert types
7	17+	advanced expert Types

Theoretical Basics

Basic Equations

$$WL = \frac{m_{MTO}}{S}$$

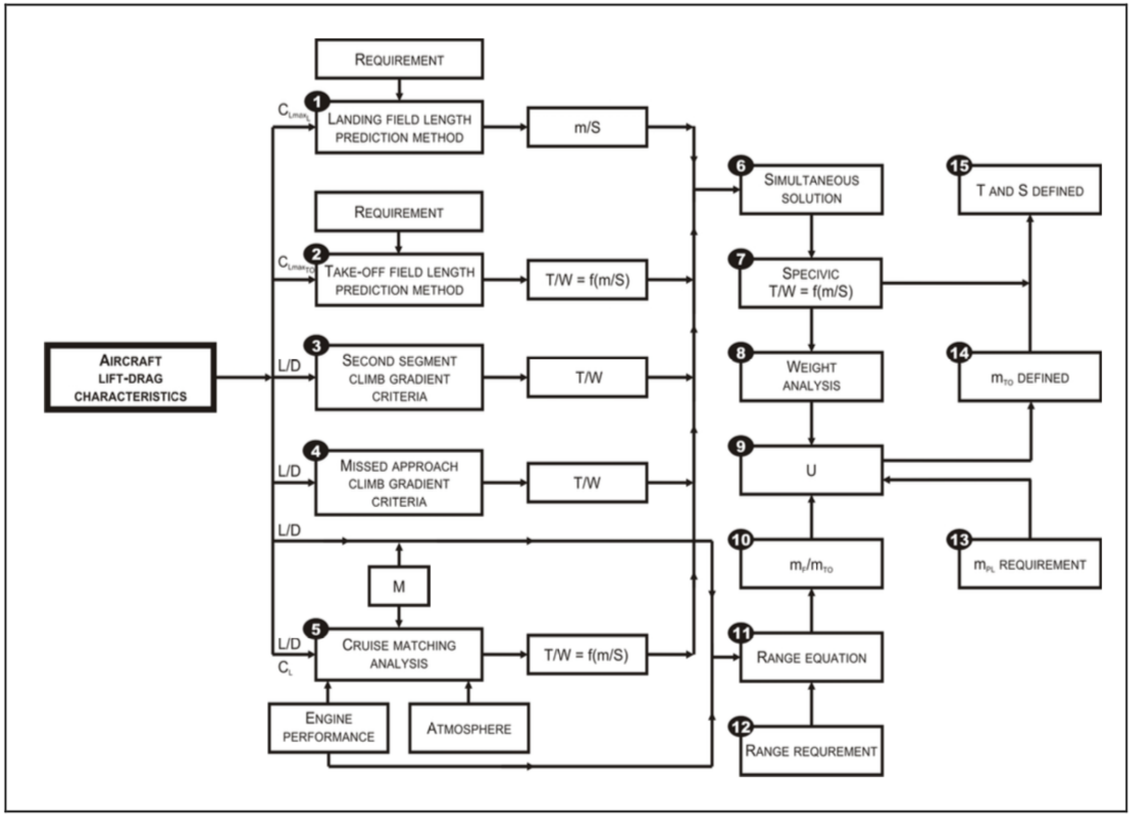
$$CWL = \frac{m_{MTO}}{S^{3/2}}$$

CWL=f(WL)	Aircraft Speed	Speed relation for 2 models
$CWL = \frac{1}{S^{0.5}} WL$	$V \,\alpha\, \sqrt{WL}$	$V_2^2 = \frac{WL_2}{WL_1} V_1^2$
$CWL = \frac{1}{m^{0.5}} WL^{1.5}$	$V \,\alpha\, \sqrt{CWL\, S^{0.5}}$	$V_2^2 = \frac{CWL_2}{CWL_1} a V_1^2$
		(a =scale factor)

- Relations become more complex

Preliminary Sizing

Preliminary Sizing



Preliminary sizing follows Chapter 5 of Aircraft Design lecture notes by D. Scholz; only necessary changes are discussed.

Preliminary Sizing Equations Introducing CWL Parameter

Preliminary Sizing

- **Landing distance**

$$\frac{m_{MTO}}{S_W} = k_L S_{LFL} \sigma C_{L,max,L} \frac{m_{MTO}}{m_{ML}} \longrightarrow \frac{m_{MTO}}{S_W^{3/2}} = \frac{m_{ML}^{3/2} / S_W^{3/2}}{m_{ML}^{3/2} / m_{MTO}} = \frac{(k_L S_{LFL} \sigma C_{L,max,L})^{3/2}}{m_{ML}^{\frac{3}{2}} / m_{MTO}}$$

- **Take-Off distance**

$$\frac{T_{TO}}{m_{MTO} g} = \frac{k_{TO}}{s_{TOFL} \sigma C_{L,max,TO}} \frac{m_{MTO}}{S_W} \longrightarrow \left(\frac{T_{TO}}{m_{MTO} g}\right)^{3/2} = \left(\frac{k_{TO}}{s_{TOFL} \sigma C_{L,max,TO}}\right)^{3/2} \sqrt{m_{MTO}} \frac{m_{MTO}}{S_W^{3/2}}$$

- **Climb rate during 2nd segment**

$$\frac{T_{TO}}{m_{MTO} g} = \frac{n_E}{n_E - 1} \left(\frac{1}{E} + \sin\gamma\right) \longrightarrow \frac{T_{TO}}{m_{MTO} g} = \frac{n_E}{n_E - 1} \left(\frac{1}{E} + \sin\gamma\right)$$

Preliminary Sizing Equations Introducing CWL Parameter

Preliminary Sizing

- **Climb rate during missed approach**

$$\frac{T_{TO}}{m_{MTO}g}=\frac{n_E}{n_E-1}\left(\frac{1}{E}+\sin\gamma\right)\frac{m_{ML}}{m_{MTO}} \quad\longrightarrow\quad \frac{T_{TO}}{m_{MTO}g}=\frac{n_E}{n_E-1}\left(\frac{1}{E}+\sin\gamma\right)\frac{m_{ML}}{m_{MTO}}$$

- **Cruise**

$$\frac{T_{TO}}{m_{MTO}g}=\frac{1}{(T_{CR}/T_{TO})E} \quad\longrightarrow\quad \frac{T_{TO}}{m_{MTO}g}=\frac{1}{(T_{CR}/T_{TO})E} \qquad \mathbf{f(h)}$$

$$\frac{m_{MTO}}{S_W}=\frac{C_L M^2}{g}\frac{\gamma_{heat}}{2}p(h) \quad\longrightarrow\quad \frac{m_{MTO}}{S_W^{3/2}}=\left(\frac{C_L\gamma}{2g}p(h)\right)^{3/2}\frac{M^3}{\sqrt{m_{MTO}}} \qquad \mathbf{f(h)}$$

Preliminary Sizing – $m_{ML}^{3/2}$ / m_{MTO}

design range classification	design range (NM)	design range (km)	$m_{ml}^{3/2}/m_{mto}$
short range	up to 1000	up to 2000	119.6
medium range	1000-3000	2000-5500	188.3
long range	3000-8000	5500-15000	289.8
ultra long range	more than 8000	more than 15000	323.7

Preliminary Sizing - m_{MTO}

$$m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

- $\frac{m_F}{m_{MTO}}$

$$\frac{m_F}{m_{MTO}} = 1 - M_{ff} \longrightarrow \frac{m_{i+1}}{m_i} = e^{-\frac{S_i}{B_s}} \longrightarrow B_s = \frac{L/_D V}{SFC_i g}$$

Iterative process is needed:

- An approximate value for T_{CR}/T_{TO} is guessed: ~ 0.1 - 0.3
- Altitude is calculated through equation: $\frac{T_{CR}}{T_{TO}} = (0{,}0013\, BPR - 0{,}0397)\frac{h_{CR}}{1000} - 0{,}0248\, BPR + 0{,}7125$
- A value for m_F/ m_{MTO} is obtained.
- Later, with m_{OE}/ m_{MTO}, m_{MTO} is calculated.
- With m_{MTO} preliminary sizing is completed.
- With the $T_{TO}/m_{MTO}g$ a new T_{CR}/T_{TO} value is calculated with: $\frac{T_{TO}}{m_{MTO}g} = \frac{1}{(T_{CR}/T_{TO})E}$
- The process continues until convergence.

Preliminary Sizing - m_{MTO}

- $\frac{m_{OE}}{m_{MTO}}$

$$\frac{m_{OE}}{m_{MTO}} = 0.23 - 1.04 \cdot \frac{T_{TO}}{m_{MTO}\, g}$$

$$\frac{m_{OE}}{m_{MTO}} = A m_{MTO}{}^{*C} k_{vs}$$

$$\frac{m_{OE}}{m_{MTO}} = k + uR$$

$$\frac{m_{OE}}{m_{MTO}} = R^{w} m_{PL}^{m} n_{e}^{n} V_{CR}^{z}$$

$$\frac{m_{OE}}{m_{MTO}} = k$$

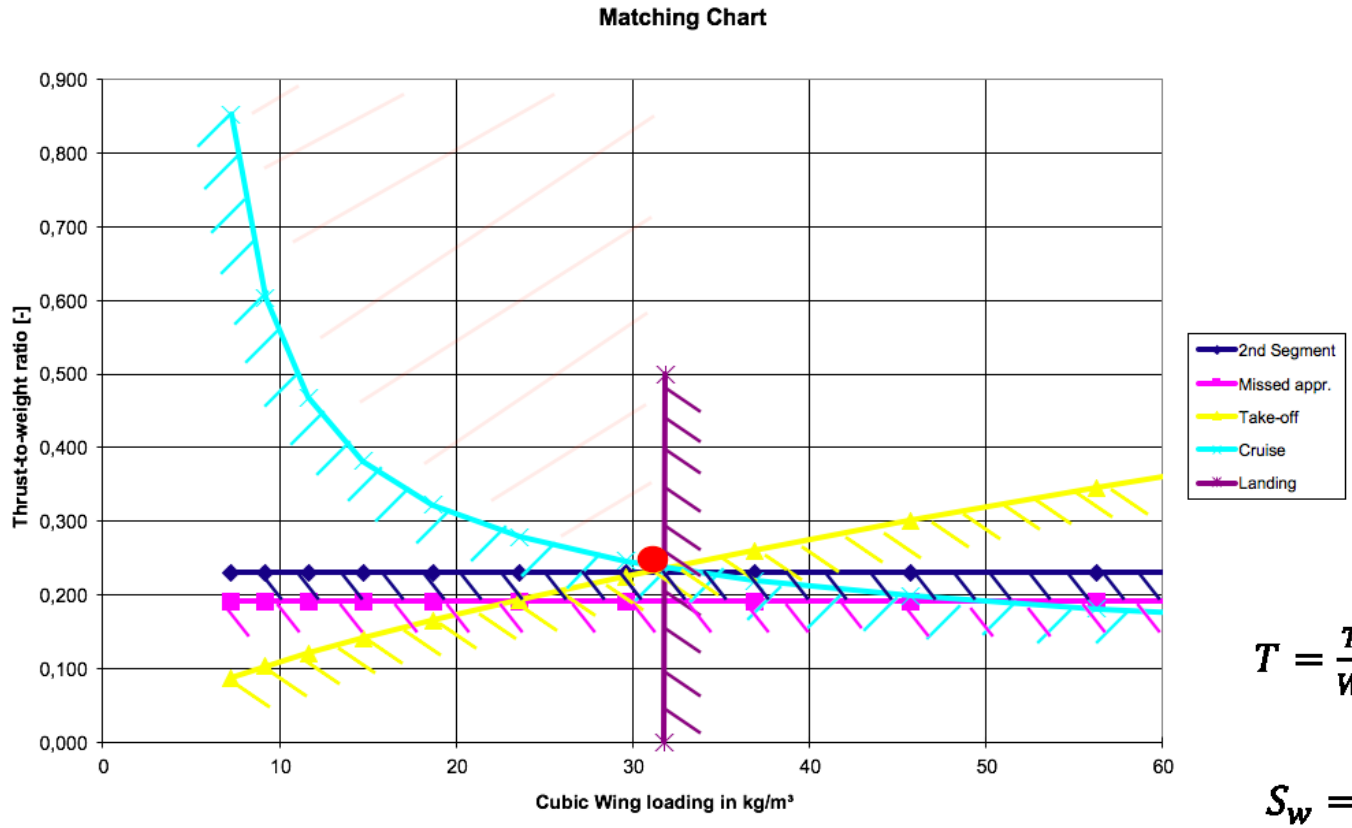
A	1.02
c	-0.06
k_{vs}	1
error	6.3%

k	0.5967
u	-1.657E-05
error	4.48%

* An iterative process is needed:

- An approximate value for m_{MTO} is guessed.
- The ratio m_{OE}/m_{MTO} is calculated.
- A new value for m_{MTO} is obtained : $m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$
- The process continues until convergence.

Preliminary Sizing - Matching Chart



$$T = \frac{T}{W} m_{MTO} g$$

$$S_w = \left(\frac{m_{MTO}}{CWL}\right)^{\frac{2}{3}}$$

Preliminary Sizing - Relative Errors

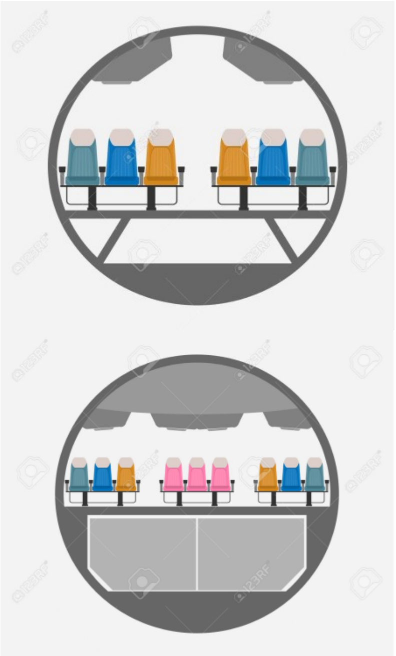
	m_{MTO} (kg)	relative error (%)	V_{md}/V
WL based preliminary sizing	254,272	-	1.36
m_{OE}/m_{MTO} acc. to Raymer	259,053	1.88	1.31
m_{OE}/m_{MTO} acc. to eq. 4.17	250,371	1.53	1.43
m_{OE}/m_{MTO} acc. to sts. (original)	255,102	0.33	1.36

Statistical Correlations

Statistical Correlations

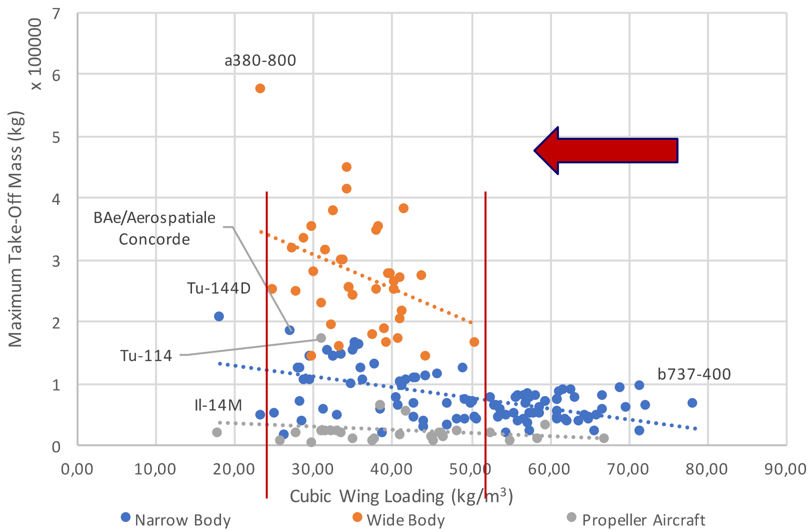
General Observations

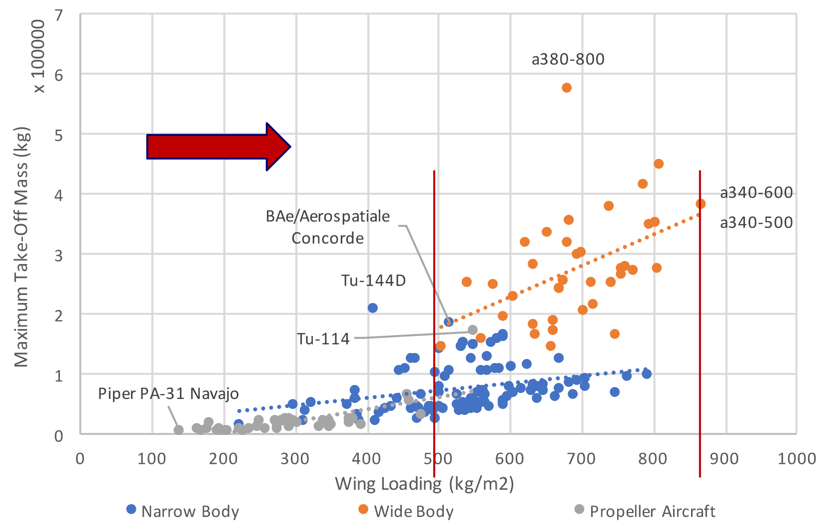
- Aircraft were divided:
    - Narrow body
    - Wide body
    - Propeller aircraft
- Out of trend planes were identified.
- The analysis considers the following parameters:
    - Maximum Take-Off Mass
    - Wing Surface
    - Aspect Ratio
    - Maximum Mach Number/Aircraft's Speed
    - Range
    - Wetted Area
    - Evolution through Years



Findings

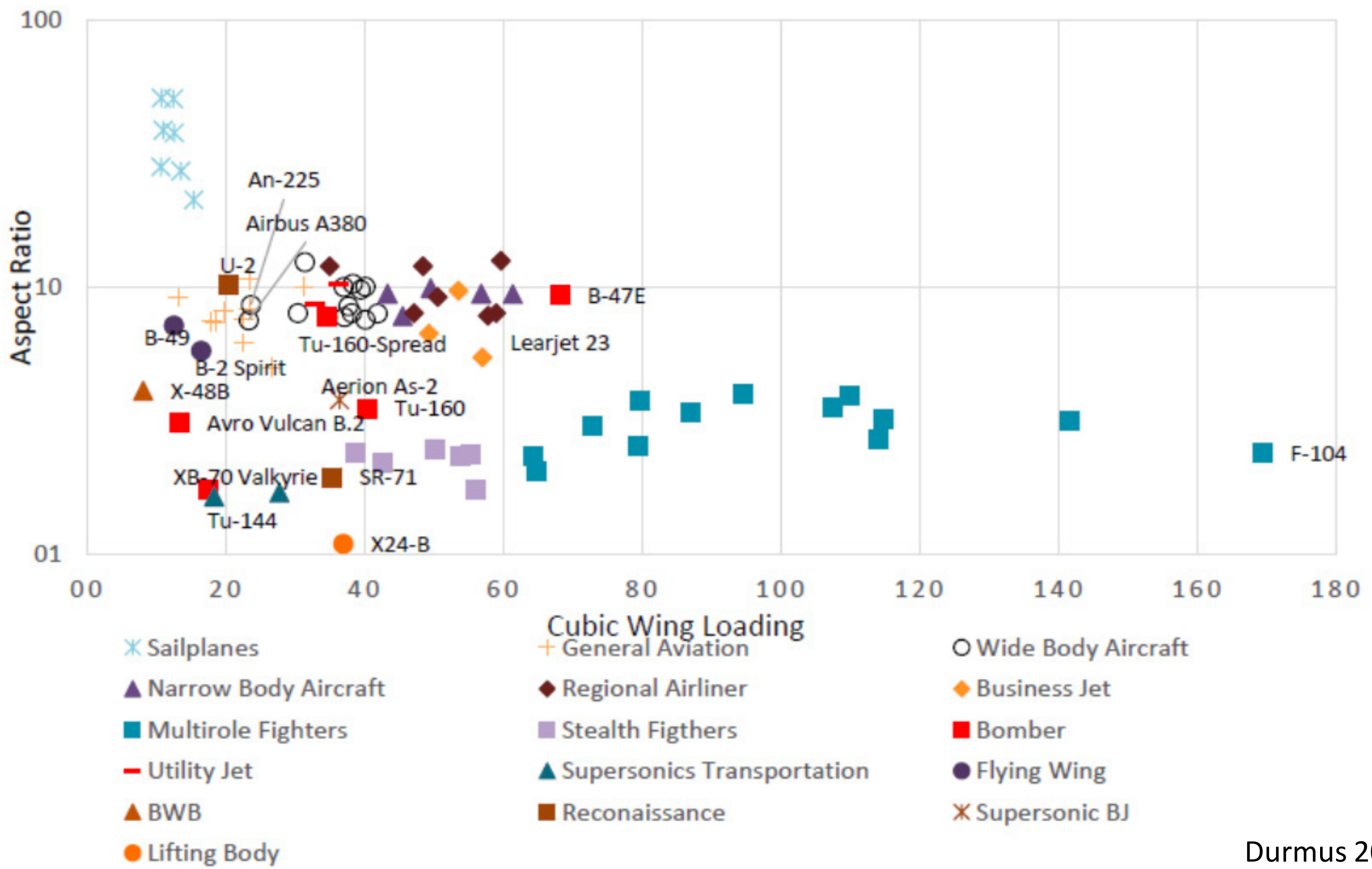
- Statistical correlations are weak.
- CWL is not a constant parameter for different aircraft.





Statistical Correlations

Findings

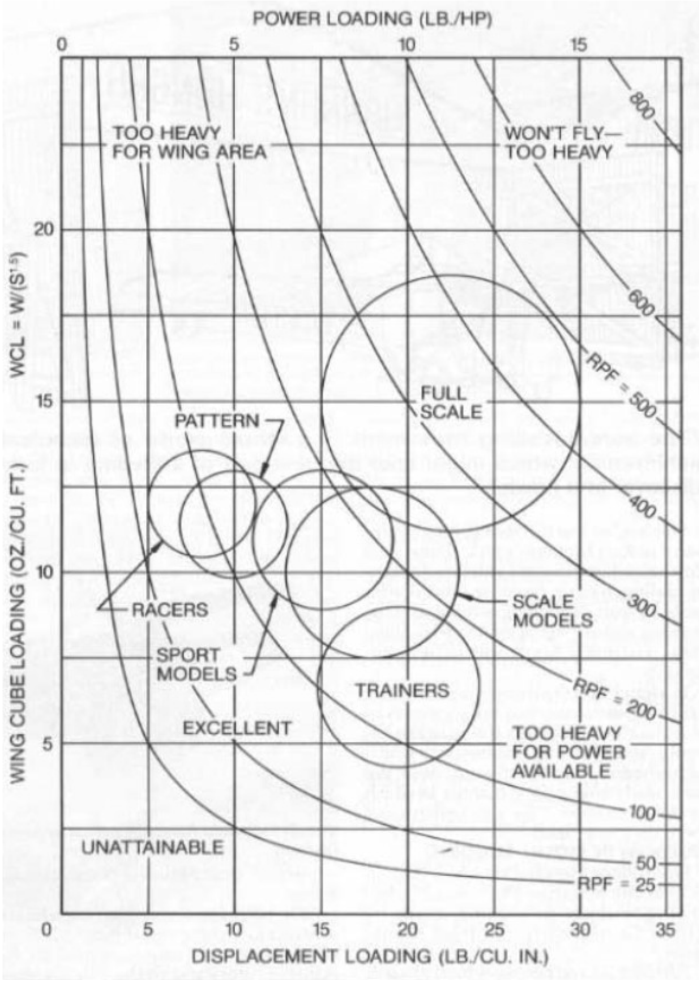


Durmus 2020

Other Relations

Other Relations

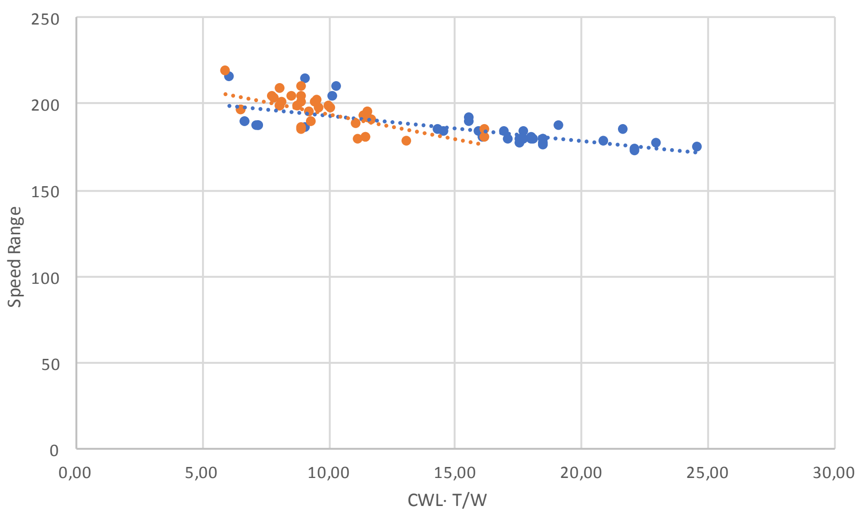
Performance Factor

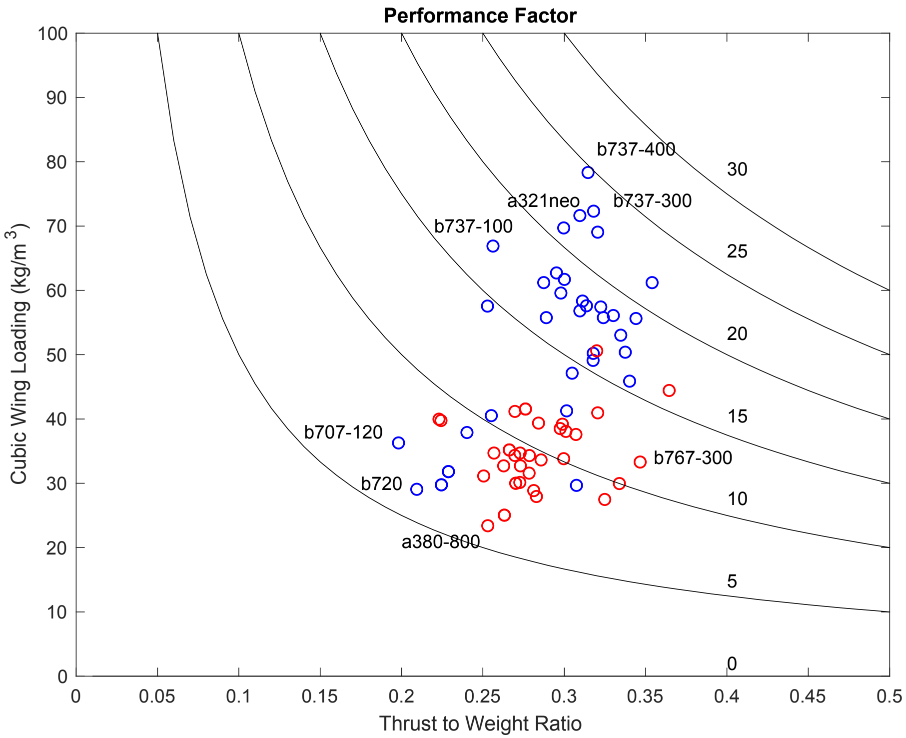


Reynolds 1989

Other Relations

Performance Factor

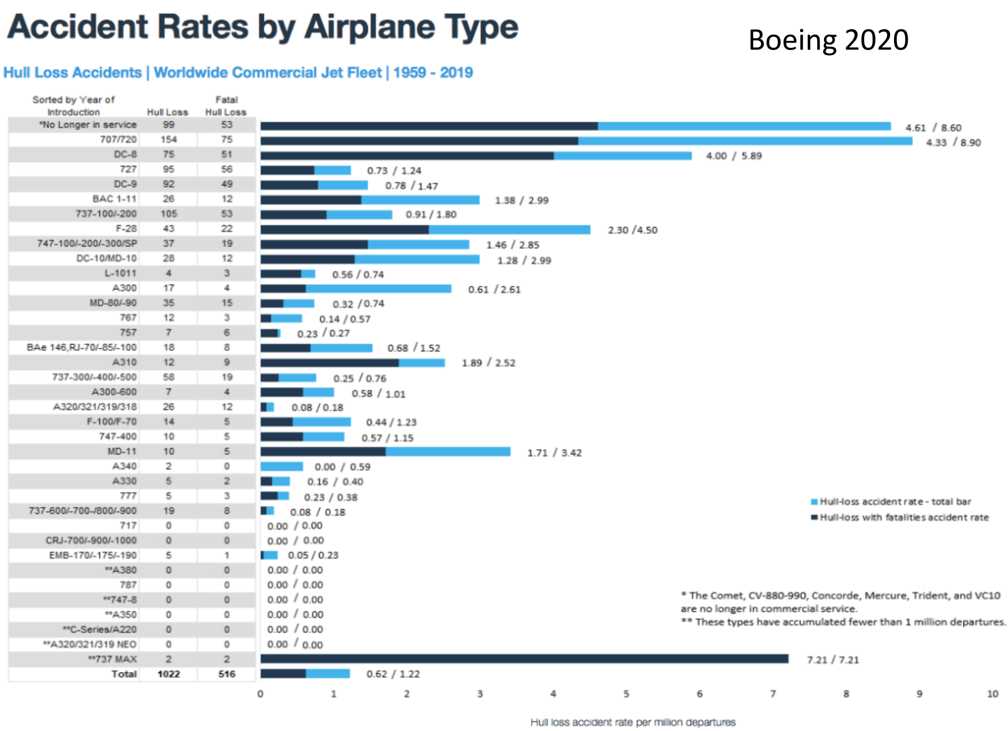


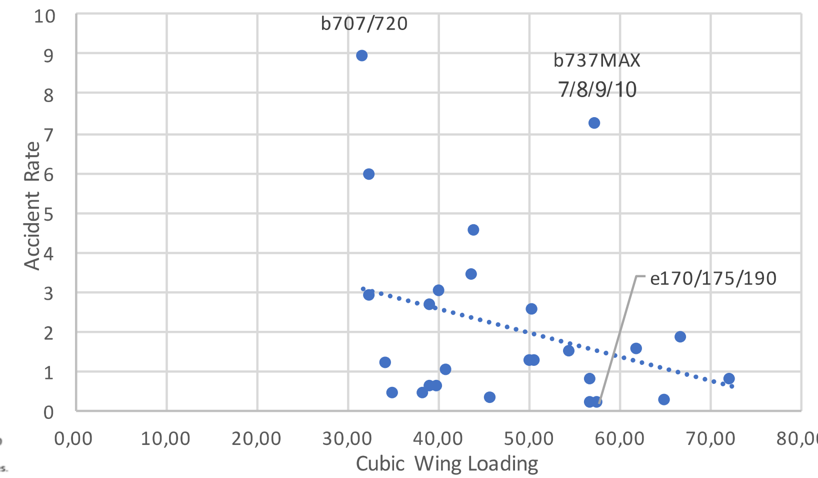


aircraft type	equation	R^2
narrow body	y = -1,4146x + 206,96	0,49984
wide body	y = -2,889x + 223,15	0,48615

Other Relations

Accident Rate



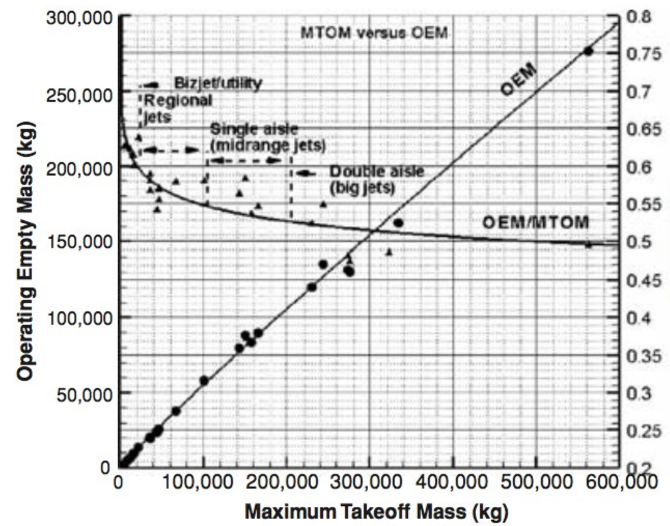


aircraft type	equation	R^2
narrow body	y = -0,1508x + 10,472	0,39723
wide body	y = 0,0465x - 0,3139	0,04351

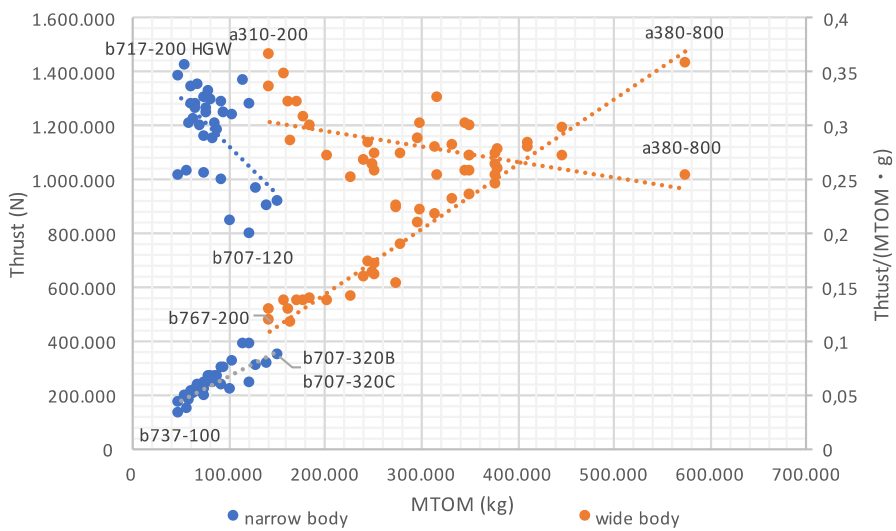
Other Relations

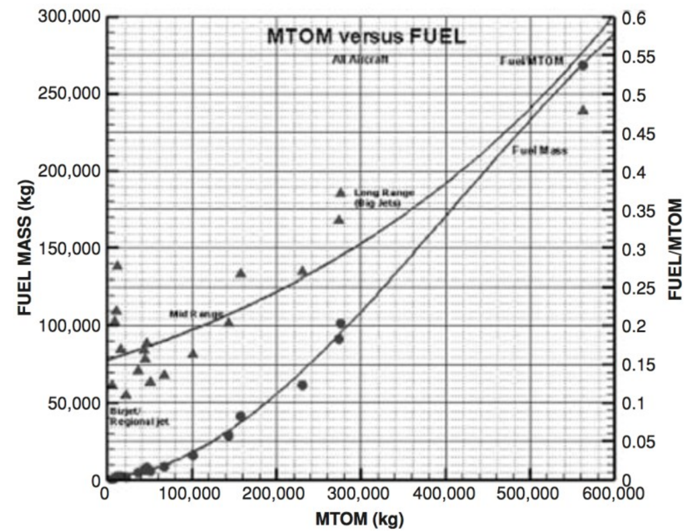
Structural and Fuel Efficiency

- Larger aircraft have better structural efficiency and worst fuel economy.
- New technology (materials) lets built larger aircraft with lower CWL.
- Lower CWL aircraft move to the left on the charts: better fuel economy and worst structural efficiency are achieved (Airbus 380).



Kundu 2010





Conclusions

Conclusions

- **CWL** is not a constant parameter for passenger aircraft.
- **Preliminary Sizing** can be done based on CWL but an iterative process is needed.
- **Relations** with other parameters are weak and equations are more complex than with traditional WL.
- **High CWL aircraft** are not related with high accident rate.
- **CWL** gives some clues about aircraft speed range, fuel economy and structural efficiency.

Summary

Summary

- In **RC controlled** planes CWL is an interesting and useful design parameter. We wanted to know if it could provide some advantages also for passenger aircraft design.

- **WL** is directly obtained from the lift equation, in contrast to **CWL**. Introducing this new parameter (with a power of 3/2) gives equation a complex form.

- **Preliminary sizing** can be done based on CWL, rewriting equations and obtaining a matching chart for T/W vs. CWL, but an iterative process is needed.

- CWL relation with other useful aircraft parameter was studied. **Statistical correlations** are weak.

- Finally, how CWL was related with **aircraft speed range**, **accident rate**, **fuel economy** or **structural efficiency** was addressed, bringing interesting findings.

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