5 Preliminary Sizing

The preliminary sizing of an aircraft is carried out by taking into account requirements and constraints (see Section 1). A process for preliminary sizing proposed by **Loftin 1980** is shown in **Fig. 5.1** and detailed in this section. The procedure refers to the preliminary sizing of jets that have to be certified to CS-25 or FAR Part 25. The procedure could in general also be applied to other aircraft categories as there are

- very light jets certified to CS-23/FAR Part 23
- propeller aircraft certified to CS-25/FAR Part 25
- propeller aircraft certified to CS-23/FAR Part 23
- propeller aircraft certified to CS-VLA
- ...

if the respective special features and regulations are taken into account. For propeller-type aircraft the engine thrust *T* must be replaced by engine power *P* in **Fig. 5.1**. Many changes in the equations result from this modification.

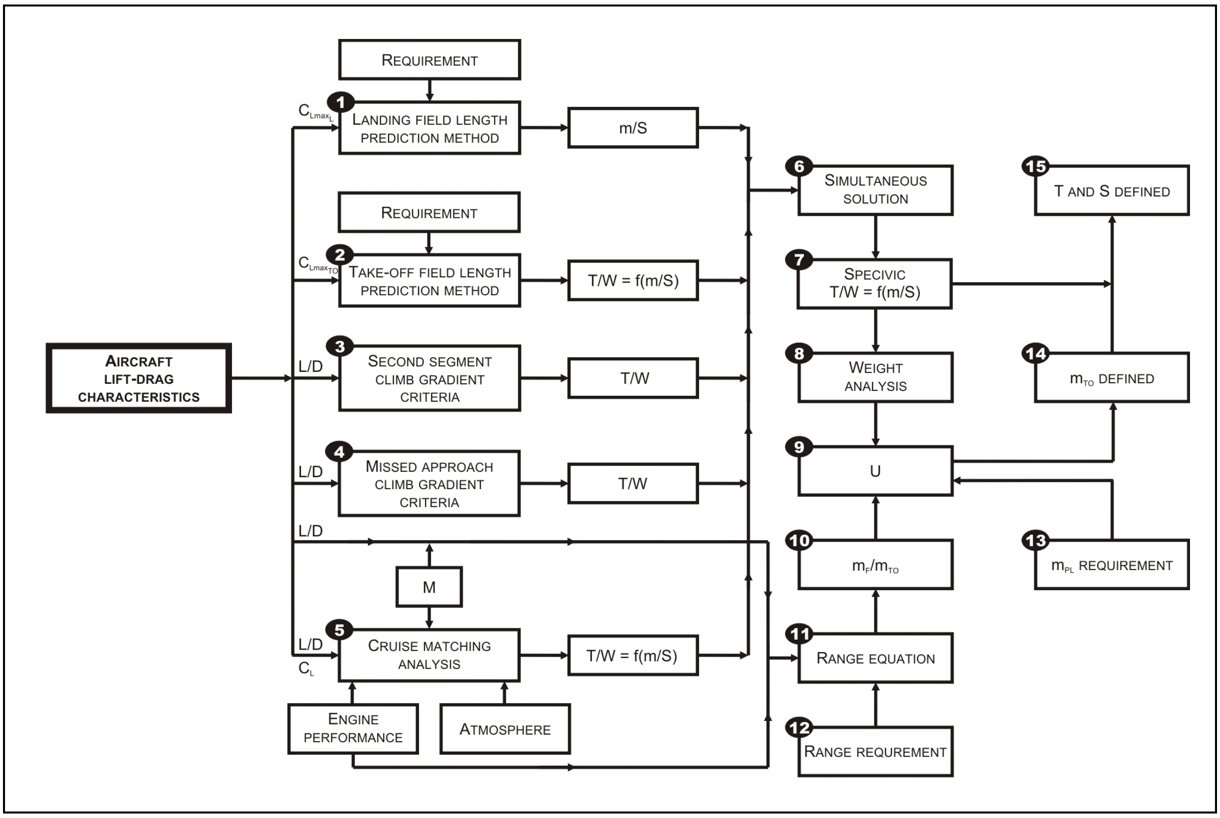


Fig. 5.1 Flow chart of the aircraft preliminary sizing process for jets based on **Loftin 1980**

Fig. 5.1 needs some explanation: The blocks in the first column represent calculations for various flight phases.

Block 1 "LANDING DISTANCE" provides a maximum value for the *wing loading* m/S (reference value: m_{MTO}/S_W). The input values of the calculation are the maximum lift coefficient with flaps in the landing position $C_{L,max,L}$ as well as the landing field length s_{LFL} according to CS/FAR. The maximum lift coefficient $C_{L,max,L}$ depends on the type of high lift system and is selected from data in the literature.

Block 2, "TAKE-OFF DISTANCE" provides a minimum value for the *thrust-to-weight ratio as a function of the wing loading*: $T/(m \cdot g) = f(m/S)$ with reference value: $T_{TO}/(m_{MTO} \cdot g)$. The functional connection $T/(m \cdot g) = f(m/S)$ is dependent on the maximum lift coefficient with flaps in the take-off position $C_{L,max,TO}$ and the take-off field length s_{TOFL}. The maximum lift coefficient $C_{L,max,TO}$ is selected with the aid of data in the literature.

Blocks 3 and 4 examine the "CLIMB RATE IN THE SECOND SEGMENT" and the "CLIMB RATE DURING THE MISSED APPROACH". The blocks provide minimum values for the *thrust-to-weight ratio* $T/(m \cdot g)$. The input value for the calculations: the lift-to-drag ratio, 'L over D') L/D is estimated on the basis of a simple approximation calculation.

Block 5 "CRUISE" represents the cruise analysis that provides a minimum value for the *thrust-to-weight ratio as a function of the wing loading*: $T/(m \cdot g) = f(m/S)$. The thrust-to-weight ratio thus determined is sufficient to facilitate a stationary straight flight with the assumed Mach number for the respective wing loading. The calculation is carried out for the design lift coefficient $C_{L,DESIGN}$. The *cruise altitude* is also obtained from the cruise analysis. Input values are the lift-to-drag ratio $E = L/D$ during cruise, the assumed cruise Mach number $M = M_{CR}$, engine parameters and the characteristics of the atmosphere.

The output values of the blocks in the first column of Fig. 5.1 provide a set of relationships between the thrust-to-weight ratio and the wing loading. Taken together, these relationships give, in a "MATCHING CHART" (**Blocks 6 and 7**), *a single pair of values: thrust-to-weight ratio and wing loading* that meets all requirements and constraints in an economical manner.

The thrust-to-weight ratio (also referred to as the maximum take-off mass or range by other authors) are the input values for a mass estimate according to statistics. In **Blocks 8 and 9,** first the OPERATING EMPTY MASS m_{OE}/m_{MTO} or the *RELATIVE USEFUL LOAD u* is estimated, defined as

$$u = \frac{m_F + m_{PL}}{m_{MTO}} = 1 - \frac{m_{OE}}{m_{MTO}} \quad . \tag{5.1}$$

The useful load is $m_F + m_{PL}$, maximum take-off mass m_{MTO} and operating empty mass m_{OE} .

In **Blocks 10 and 11** the RELATIVE FUEL MASS m_F / m_{MTO} is calculated, using the "Breguet range equation", from the range requirement (**Block 12**). Other input values are the assumed cruise Mach number $M = M_{CR}$, the lift-to-drag ratio during cruising $E = L/D$ and the specific fuel consumption during cruising $c = SFC_{CR}$.

"MAXIMUM TAKE-OFF MASS m_{MTO}", "TAKE-OFF THRUST AND WING AREA": in **Block 14** the *maximum take-off mass* m_{MTO} is calculated from relative useful load *u*, relative fuel mass m_F / m_{MTO} and the *payload requirement* m_{MPL} (**Block 13**). With the maximum take-off mass m_{MTO} the necessary *take-off thrust* $T = T_{TO}$ and the *wing area* $S = S_W$ can then be calculated in **Block 15** from thrust-to-weight ratio $T / m \cdot g$ and wing loading $S = S_W$.

5.1 Landing Distance

The basis for analyzing the landing distance are the aviation regulations. The key passages are reproduced here from CS. Further details can be found in the regulations.

CS 25.125 Landing
(a) The horizontal distance necessary **to land** and to come to a complete stop **from a point 50 ft above the landing surface** must be determined
(1) The aeroplane must be in the landing configuration.
(2) A stabilised **approach**, with a calibrated **airspeed** of not less than **1·3 VS**, must be maintained down to the 50 ft height.

CS - OPS 1.515 Landing - Dry Runways
(a) An operator shall ensure that the landing mass of the aeroplane ... allows a full stop landing from 50 ft above the threshold:
(1) Within **60% of the landing distance available** at the destination aerodrome and at any alternate aerodrome for **turbojet** powered aeroplanes; or
(2) Within **70% of the landing distance available** at the destination aerodrome and at any alternate aerodrome for **turbopropeller** powered aeroplane ...

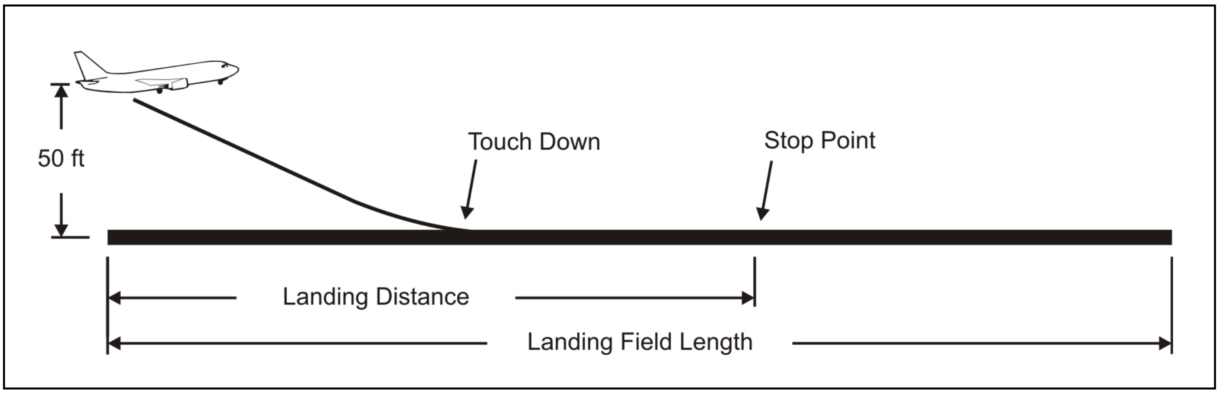


Fig. 5.2 Definition of the landing field length according to CS and FAR

An aircraft may land at an airfield if the *landing field length* s_{LFL} is shorter than the *landing distance available*, LDA s_{LDA}. The landing field length is calculated according to CS/FAR from the landing distance s_L and a **safety factor**. This safety factor is 1/0.6 = **1.667** for **jets** and 1/0.7 = **1.429** for **turboprops**. **Fig. 5.2** shows the landing procedure.

Loftin 1980 contains a statistic that gives the relationship between the landing field length and the approach speed for aircraft with jet engines. This is summarized as follows (**Loftin 1980,** Fig. 3.4**)**

$$V_{APP} = k_{APP} \cdot \sqrt{s_{LFL}} \tag{5.2}$$

with $k_{APP} = 1.70 \sqrt{\mathrm{m/s^2}}$.

The wing loading at maximum landing mass is

$$m_{ML} / S_W = \frac{\rho \cdot V_{S,L}^2}{2 \cdot g} \cdot C_{L,max,L} \quad . \tag{5.3}$$

ρ is the atmospheric density. To simplify subsequent calculations, we want to relate the atmospheric density ρ to the atmospheric density at sea level ρ_0=1.225 kg/m³ under standard atmospheric conditions first of all.

$$\rho = \sigma \cdot \rho_0 \quad . \tag{5.4}$$

If we now insert equation (5.2) and equation (5.4) into equation (5.3), we get

$$m_{ML} / S_W = k_L \cdot \sigma \cdot C_{L,max,L} \cdot s_{LFL} \tag{5.5}$$

with k_L = 0.107 kg/m³ .

Empirical values for maximum lift coefficients $C_{L,max}$ are contained in **Table 5.1**, **Fig. 5.3** and **Fig. 5.4**. **Table 5.3** and **Table 5.2** contain the ratio of maximum landing mass m_{ML} to maximum take-off mass m_{MTO} , which gives

$$m_{MTO} / S_W = \frac{m_{ML} / S_W}{m_{ML} / m_{MTO}} \quad . \tag{5.6}$$

This **wing loading must not be exceeded** if the aircraft is to meet requirements.

Table 5.1 Maximum lift coefficients for take-off (TO), landing (L) and cruise configuration (based on **Roskam I**)

type of aircraft	$C_{L,max}$	$C_{L,max,TO}$	$C_{L,max,L}$
business jet	1.4 – 1.8	1.6 – 2.2	1.6 – 2.6
jet transport	1.2 – 1.8	1.6 – 2.2	1.8 – 2.8
single engine propeller driven	1.3 – 1.9	1.3 – 1.9	1.6 – 2.3
twin engine propeller driven	1.2 – 1.8	1.4 – 2.0	1.6 – 2.5
fighter	1.2 – 1.8	1.4 – 2.0	1.6 – 2.6
supersonic cruise	1.2 – 2.8	1.6 – 2.0	1.8 – 2.2

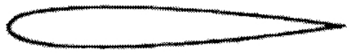
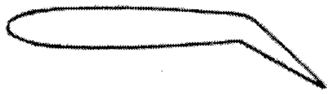
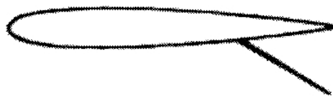
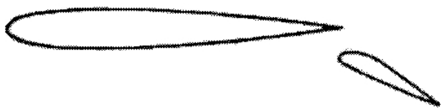
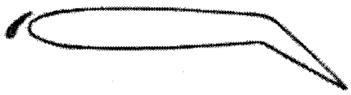
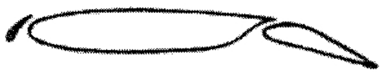
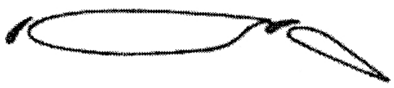
		$C_{L,max}$	$\Delta C_{L,max}$
Clean Airfoil		1,45	-
Plain Flap		2,25	0,80
Single-Slotted Flap		2,60	1,15
Double-Slotted Flap		2,80	1,35
Split Flap		2,40	0,95
Double-Wing (Junkers)		2,25	0,80
Fowler Flap		2,80	1,35
Slat		2,00	0,55
Combinations:			
Plain Flap and Slat		2,45	1,00
Single-Slotted Flap and Slat		2,70	1,25
Double-Slotted Flap and Slat		2,90	1,45
Fowler Flap and Slat		3,00	1,55

Fig. 5.3 Maximum lift coefficient of profiles with different high-lift devices (based on data from **Dubs 1987**)

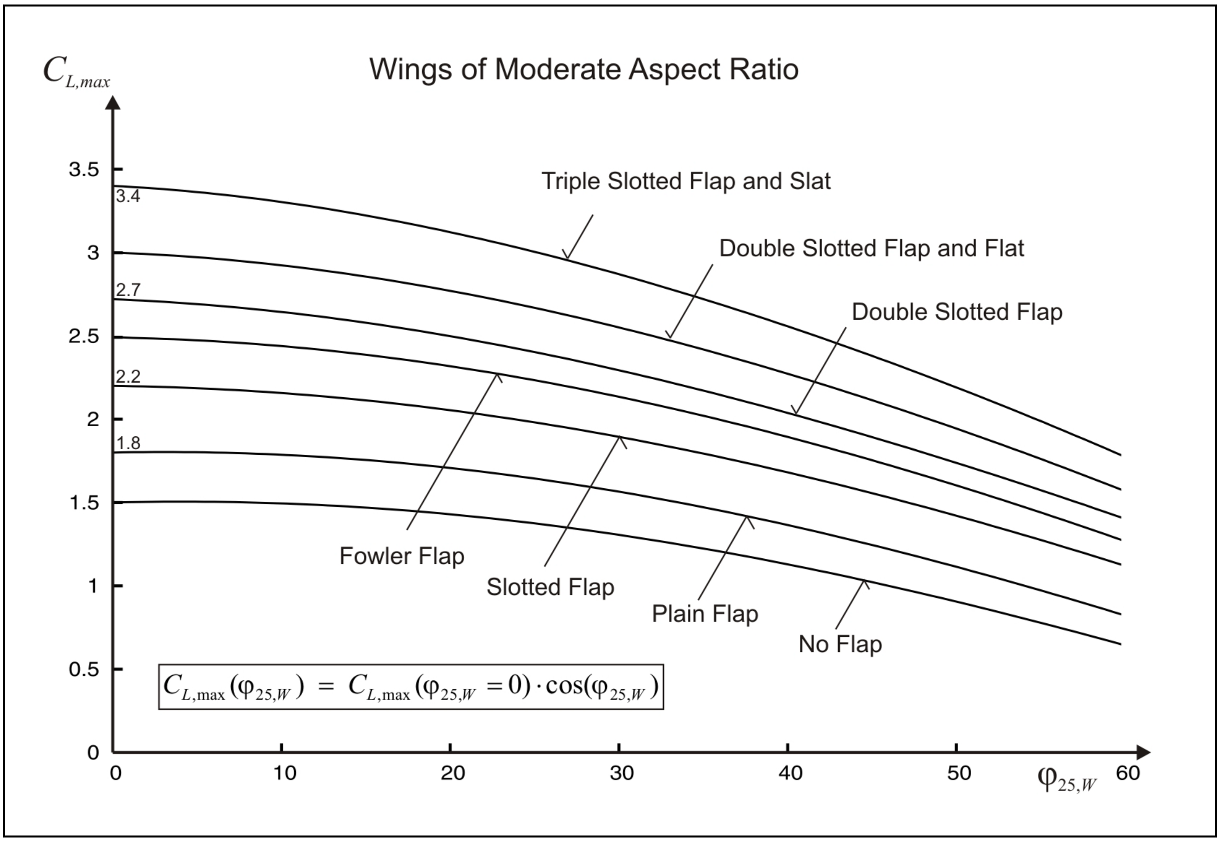


Fig. 5.4 Maximum lift coefficient of aircraft with different high-lift devices as a function of wing sweep. For take-off configuration the given values have to be reduced by 20 percent (based on data from **Raymer 1989**)

Table 5.2 Statistical values of maximum landing mass over maximum take-off mass m_{ML} / m_{MTO} for different types of aircraft (based on **Roskam I**)

type of aircraft	$\frac{m_{ML}}{m_{MTO,min}}$	$\frac{m_{ML}}{m_{MTO,av}}$	$\frac{m_{ML}}{m_{MTO,max}}$
business jet	0.69	0.88	0.96
short range jet transport	0.9	0.93	0.97
medium range jet transport	0.76	0.88	0.95
long range jet transport	0.65	0.78	0.95
ultra long range jet transport	0.65	0.71	0.73
fighter	0.57	-	1
supersonic cruise	0.63	0.75	0.88

Table 5.3 Statistical values of maximum landing mass over maximum take-off mass m_{ML} / m_{MTO} for jets of different design range (based on **Loftin 1980**)

design range classification	**design range (NM)**	**design range (km)**	m_{ML} / m_{MTO}
short range	up to 1000	up to 2000	0.93
medium range	1000 – 3000	2000 – 5500	0.89
long range	3000 – 8000	5500 – 15000	0.78
ultra long range	more than 8000	more than 15000	0.71

5.2 Take-off Distance

The basis for analyzing the take-off distance are the aviation regulations. The key passages are reproduced here according to CS-25. Further details can be found in the regulations.

CS 25.113 **Take-off distance** and take-off run
(a) Take-off distance is **the greater of** -
(1) The horizontal **distance** along the take-off path from the start of the take-off **to** the point at which the aeroplane is **35 ft above the take-off surface**, determined under CS 25.111 (d.h. mit ***Triebwerksausfall*** und Geschwindigkeit V2); or
(2) **115% of** the horizontal **distance** along the take-off path, **with *all engines operating***, from the start of the take-off **to** the point at which the aeroplane is **35 ft** above the take-off surface, as determined by a procedure consistent with CS 25.111.

CS 25.111 **Take-off path**
(a) ...
(2) **The aeroplane must be accelerated** on the ground to VEF, at which point **the critical engine must be made inoperative** and remain inoperative for the rest of the take-off; and
(3) After reaching VEF, the aeroplane must be accelerated to V2.
(b) During the acceleration to speed V2, the nose gear may be raised off the ground ... However, landing gear retraction may not be begun until the aeroplane is airborne.
(c) During the take-off path determination in accordance with sub-paragraphs (a) and (b) of this paragraph -
(2) The **aeroplane must reach V2 before it is 35 ft above the take-off surface ***

CS 25.109 **Accelerate-stop distance**
(a) The accelerate-stop distance is ...
(2) The **sum of the distances necessary to -**
(i) **Accelerate the aeroplane** from a standing start to V1 and continue the acceleration for 2·0 seconds after V1 is reached with all engines operating; **and**
(ii) **Come to a full stop** from the point reached at the end of the acceleration period prescribed in sub-paragraph (a)(2)(i) of this paragraph, assuming that the pilot does not apply any means of retarding the aeroplane until that point is reached ...

* nach CS 25.107 (take-off speeds) muss V2 auf jeden Fall größer sein als 1.2 VS.

Should an engine fail during take-off **before** the take-off decision speed V_1 has been reached, the pilot must reject take-off and brake. The distance from the take-off point to the point at which the aircraft comes to a standstill again is the *accelerate-stop distance* and must be shorter than the *accelerate-stop distance available*, ASDA.

If the pilot notices an engine failure **after** the take-off decision speed V_1 has already been exceeded, she must continue the take-off with the remaining engine(s). This results in the *take-off distance OEI,* which must be shorter than the *take-off distance available*, TODA. OEI stands for *one engine inoperative*.

If engine failure is noticed precisely when the take-off decision speed V_1 has been reached, the pilot has both possibilities, namely either to continue the take-off or to reject the take-off.

The take-off decision speed V_1 can be set arbitrarily, but there is only one take-off decision speed V_1 where the following applies:

Accelerate-stop distance = take-off distance OEI.

The take-off distance produced from meeting this condition is called *balanced field length*. **Fig. 5.5** shows the take-off procedure without *clearway* and without *stopway*.

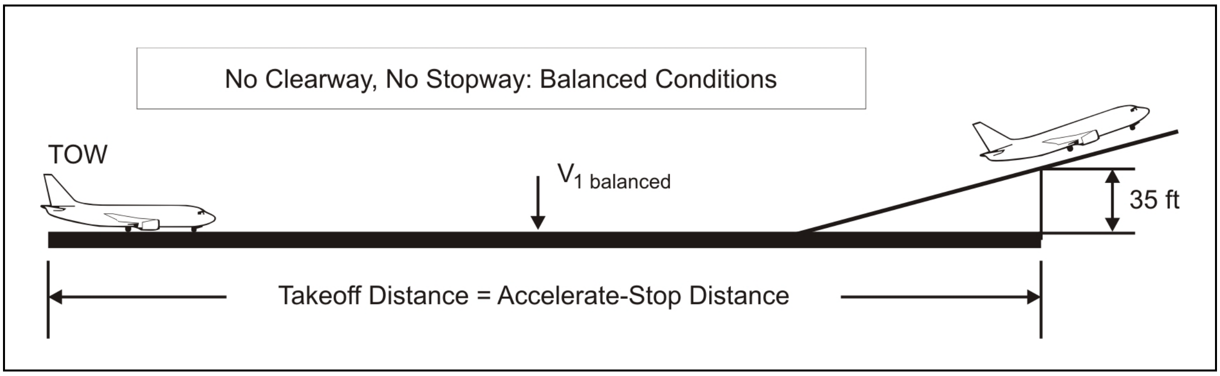


Fig. 5.5 Definition of the balanced field length according to CS and FAR (engine failure after V_1)

According to CS 25.113 (a)(2), the take-off distance AEO is 115% of the distance required to fly over an obstacle of 35ft. AEO stands for *all engines operating*. It must be shorter than the *take-off distance available*, TODA. The *take-off field length* s_{TOFL} is the larger distance in a comparison of *balanced field length* and *take-off distance AEO*.

Assuming that the thrust T, air resistance D and lift L are constant during take-off, the following applies to the *take-off ground roll*[1]:

$$s_{TOG} = \frac{1}{2} \cdot \frac{m_{TO} \cdot \left(V_{LOF} - V_W\right)^2}{T_{TO} - D_{TO} - \mu \cdot \left(m \cdot g - L_{TO}\right) - m_{TO} \cdot g \cdot \sin\gamma} \quad . \tag{5.7}$$

V_{LOF}	Lift-off speed, $V_{LOF} \approx V_2 \approx 1.2 \cdot V_{S,TO}$
$V_{S,TO}$	Stall speed in take-off configuration
V_W	Wind speed
μ	Rolling friction
γ	Runway slope

Equation (5.7) is simplified to make it usable for the aircraft design. First we calculate the *lift-off speed* from the formula $m_{TO} \cdot g = L = \frac{\rho}{2} V_{LOF}^2 \cdot C_{L,LOF} \cdot S_W$

[1] See "Flight Mechanics" lecture

$$V_{LOF} = \sqrt{\frac{2g}{\rho} \cdot \frac{m_{TO}}{S_W} \cdot \frac{1}{C_{L,LOF}}} \quad . \tag{5.8}$$

The following assumptions are made:

- V_{LOF} is only slightly less than V_2. Therefore, we calculate $V_{LOF} = 1.2 \cdot V_{S,TO}$.
- The take-off takes place on a level runway with no wind.
- The thrust T is much greater than resistance D and rolling friction.

We are taking into consideration the assumptions and insert equation (5.8) in equation (5.7) and obtain a *simplified equation for the take-off ground roll*:

$$s_{TOG} = \frac{g \cdot m_{MTO}^2}{\rho \cdot C_{L,LOF} \cdot S_W \cdot T} = \frac{1}{\rho \cdot C_{L,LOF}} \cdot \frac{m_{MTO} / S_W}{T_{TO} / (m_{MTO} \cdot g)} \quad . \tag{5.9}$$

This equation provides values which are too small for the take-off ground roll, as the drag has been ignored. However, the equation is suitable as a basis for statistical evaluations: it is assumed that the take-off field length s_{TOFL} is proportional to the take-off ground roll s_{TOG}. Furthermore, the lift coefficient $C_{L,LOF}$ is replaced by the maximum lift coefficient with flaps in take-off position $C_{L,max,TO}$. In a statistical evaluation (**Loftin 1980**, Fig. 3.7) for aircraft with jet engines the following is produced in conjunction with equation (5.4)

$$\frac{T_{TO} / (m_{MTO} \cdot g)}{m_{MTO} / S_W} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma \cdot C_{L,max,TO}} \tag{5.10}$$

with $k_{TO} = 2.34\ \text{m}^3/\text{kg}$

Table 5.1 contains values for the maximum lift coefficient with flaps in take-off position $C_{L,max,TO}$. The **ratio from thrust-to-weight ratio and wing loading** pursuant to equation 5.10 **must not be undershot** if the aircraft is to meet requirements.

5.3 Climb Rate during 2nd Segment

The take-off path is defined in several paragraphs of the certification regulations. The climb path is shown clearly in **Fig. 5.6**. The key passages regarding requirements in the second segment are quoted here according to CS-25. Further details can be found in the regulations.

> **CS 25.121 Climb: one-engine-inoperative**
> (b) *Take-off; **landing gear retracted***.
> In the take-off configuration existing at the point of the flight path at which the landing gear is fully retracted, ... the **steady gradient of climb** may **not be less than**
> **2·4%** for two-engined aeroplanes,
> **2·7%** for three-engined aeroplanes and
> **3·0%** for four-engined aeroplanes,
> at V2 and with -
> (1) The critical engine inoperative and the remaining engines at the available maximum continuous power or thrust; and
> (2) The weight equal to the weight existing at the end of the take-off path ...

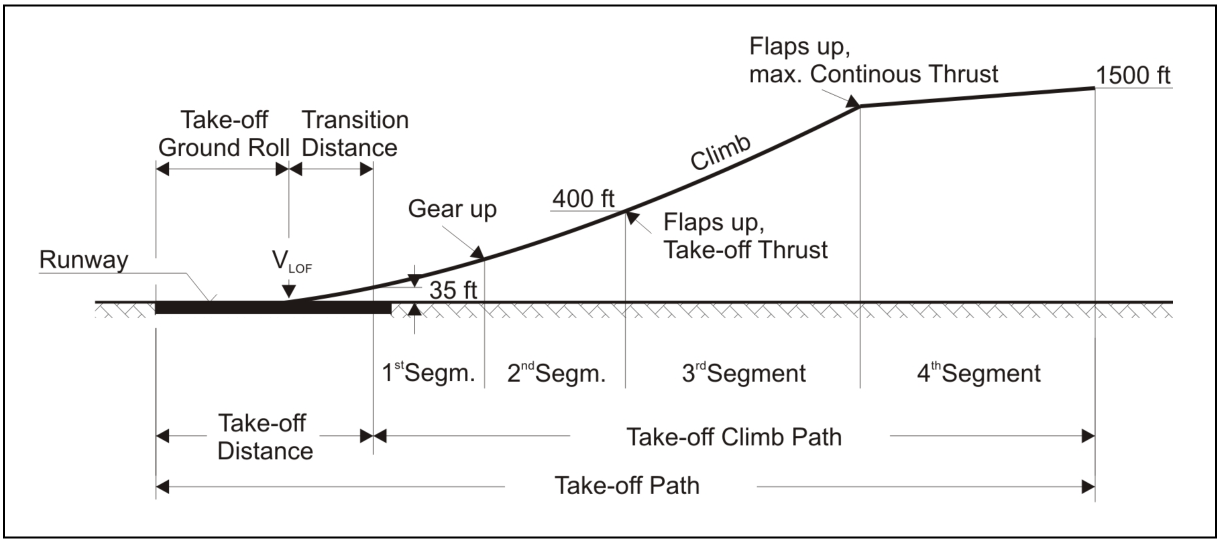


Fig. 5.6 Take-off path, definitions and nomenclature (based on **Brüning 1993**)

In the climb with climb angle γ thrust T is required to overcome drag D and weight $m \cdot g$. The following equation gives the sum of the forces in the flight direction

$$T = D + m \cdot g \cdot \sin\gamma \quad . \tag{5.11}$$

In addition, the following equation gives the force balance vertical to the flight direction (with simplification for small climb angle)

$$L = m \cdot g \cdot \cos\gamma \approx m \cdot g \quad . \tag{5.12}$$

Equation (5.11) divided by $m \cdot g$ and equation (5.12) gives

$$\frac{T}{m \cdot g} = \frac{1}{E} + \sin\gamma \quad . \tag{5.13}$$

If the climb is also to be possible with a failed engine, the thrust-to-weight ratio – *relative to the thrust of all the engines* – must be correspondingly greater. For a number of n_E engines, **at least** a **thrust-to-weight ratio of**

$$\boxed{\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E} + \sin\gamma\right)} \tag{5.14}$$

must be stipulated.

The climb angle is used in equation (5.14). However, in the regulations, the climb gradient is stated as a percentage. A conversion is simple. As

$$\tan\gamma = \frac{climb\ gradient}{100} \tag{5.15}$$

follows

$$\gamma = \arctan\frac{climb\ gradient}{100} \tag{5.16}$$

where *climb gradient* means the *value from the regulations as a percentage*. In the present calculations, the angle is small and so one can dispense with the task of conversion and directly insert the value from the regulation (e.g. for 3 % climb rate, insert 0.03) in equation (5.14), as

$$\boxed{\sin\gamma \approx \frac{climb\ gradient}{100}} \tag{5.17}$$

5.4 Lift-to-Drag Ratio with Extended Landing Gear and Extended Flaps

In equation (5.14) the **lift-to-drag ratio** $E = L/D$, which is to be **calculated** here **with an approximation procedure**, is still unknown. It is

$$E = L/D = \frac{C_L}{C_D} \tag{5.18}$$

The drag is comprised of profile drag and induced drag. The induced drag depends on the lift coefficient, the wing aspect ratio and Oswald's efficiency factor

$$C_D = C_{D,P} + \frac{C_L^{\,2}}{\pi \cdot A \cdot e} \tag{5.19}$$

$$E = \frac{C_L}{C_{D,P} + \dfrac{C_L^{\,2}}{\pi \cdot A \cdot e}} \quad . \tag{5.20}$$

The profile drag is comprised of the zero-lift drag and the additional drags due to the high lift system and, if applicable, the landing gear.

$$C_{D,P} = C_{D,0} + \Delta C_{D,flap} + \Delta C_{D,slat} + \Delta C_{D,gear} \quad . \tag{5.21}$$

The approximation procedure according to **Loftin 1980** applied to normal passenger aircraft makes the following assumptions to estimate lift-to-drag ratio:

e	0.7 due to extended flaps and slats
$C_{D,0}$	0.02
$\Delta C_{D,flap}$	for $C_L = 1.3$: flaps 15° => $\Delta C_{D,flap} = 0.01$
	for $C_L = 1.5$: flaps 25° => $\Delta C_{D,flap} = 0.02$
	for $C_L = 1.7$: flaps 35° => $\Delta C_{D,flap} = 0.03$
$\Delta C_{D,slat}$	negligible
$\Delta C_{D,gear}$	0.015 in case landing gear is extended.

The *maximum* lift coefficients in the case of the three stated flap positions are naturally higher. In the climb after take-off at $V_2 = 1.2 \cdot V_{S,TO}$, the $C_{L,max,TO} = 1.44 \cdot C_L$ and during the missed approach after the landing approach at $V_{MA} = 1.3 \cdot V_{S,L}$, the $C_{L,max,L} = 1.69 \cdot C_L$. In this case the procedure is such that the following conversion is used to estimate the maximum lift coefficient from the predefined maximum lift coefficients:

$$C_L = C_{L,max} \left(\frac{V_s}{V} \right)^2 \tag{5.21a}$$

The values according to **Loftin 1980** for $\Delta C_{D,flap}$ can also be summarized in a formula

$$\boxed{\begin{gathered} \Delta C_{D,flap} = 0.05\, C_L - 0.055 \qquad (5.21b) \\ \text{for } C_L \geq 1.1 \quad . \end{gathered}}$$

5.5 Climb Rate during Missed Approach

During a missed (discontinued) approach the aircraft is in the process of making the final approach. For some reason a decision is taken not to land. Take-off thrust is applied, the aircraft climbs and makes a new approach according to a predefined procedure. The aircraft climbs, although it is still in the landing configuration – with considerable drag: the landing gear has already been extended and the flaps are in landing position. The regulations require sufficient installed thrust to carry out this maneuver safely. The key passages relating to requirements for the missed approach are quoted here according to CS-25. Further details can be found in the regulations.

CS 25.121 Climb: one-engine-inoperative
(d) **Discontinued Approach.** ... the **steady gradient** may **not be less than**
2·1% for two-engined aeroplanes,
2·4% for three-engined aeroplanes and
2·7% for four-engined aeroplanes, with -
(1) **The critical engine inoperative**, the remaining engines at the available take-off power or thrust;
(2) The maximum landing weight; and
(3) A climb speed established in connection with normal landing procedures (these are 1·3 VS), but not exceeding 1·5 VS.
(4) Landing gear retracted. *

* (4) is only contained in CS-25 <u>not</u> in den FAR Part 25 !!!

The calculation method for the missed approach is very similar to the method used for the second segment. When estimating the lift-to-drag ratio $E = L/D$ it must be borne in mind that (according to FAR Part 25, but not according to CS-25!!!) the landing gear is still extended. The necessary thrust-to-weight ratio is

$$\frac{T_{TO}}{m_{ML} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E} + \sin\gamma\right) \tag{5.22}$$

and in this case relates initially to the maximum landing mass. However, as all the calculations in the matching chart (Blocks 6 and 7) use parameters which relate to take-off, the thrust-to-weight ratio has to be converted to the maximum take-off mass.

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{T_{TO}}{m_{ML} \cdot g} \cdot \frac{m_{ML}}{m_{MTO}} \quad . \tag{5.23}$$

For the missed approach the equation to determine the **minimum value of the thrust-to-weight ratio** is as follows:

$$\boxed{\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E} + \sin\gamma\right) \cdot \frac{m_{ML}}{m_{MTO}}} \tag{5.24}$$

5.6 Cruise

For calculations in the cruise phase, a **stationary straight flight** at cruise altitude is assumed. Therefore two equations can be used:

a) lift is equal to weight;
b) drag is equal to thrust.
With these two equations a statement can then be made on:
a) the wing loading;
b) the thrust-to-weight ratio.

The connection between the **wing loading** and the **thrust-to-weight ratio** is determined in such a way that both parameters are **initially calculated separately** as a function of altitude. The connection between the two parameters is then obtained automatically from the individual results via the connection with the altitude.

5.6.1 Thrust-to-Weight Ratio

In cruise flight – i.e. in a stationary straight flight – the following applies to the thrust T_{CR} and the drag D_{CR}

$$T_{CR} = D_{CR} = \frac{m_{MTO} \cdot g}{E} \quad . \tag{5.25}$$

Strictly speaking, the performance requirement in cruise flight is that of a climb. The reason for this is the definition of the service ceiling. The definition states that when flying at service ceiling, a jet still has to reach a climb speed of 500 ft/min. Accordingly, for flights at any other, lower altitude, at least the same climb speed of 500 ft/min would be expected. The formula $T_{CR} = D_{CR}$ is therefore not a conservative estimate, but has the advantage of producing a simple equation. However, this is balanced out in (5.25) by the fact that the maximum take-off mass is assumed as the aircraft mass. The actual mass in cruise flight is less than m_{MTO} due to the consumption of fuel since take-off. At this point this leads to a small safety margin. It is assumed that this safety margin balances out the non-conservative assumption $T_{CR} = D_{CR}$ for the cruise flight.

The equation (5.25) is divided by the take-off thrust T_{TO}. This gives

$$\frac{T_{CR}}{T_{TO}} = \frac{m_{MTO} \cdot g}{T_{TO} \cdot E} \tag{5.26}$$

or

$$\boxed{\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{\left(T_{CR} / T_{TO}\right) \cdot E} \quad . \tag{5.27}}$$

Lift-to-drag ration E is estimated from the wing aspect ratio, as is explained below in Section 5.7.

T_{CR} / T_{TO} can be read off engine diagrams for a given altitude and Mach number. For normal cruise Mach numbers of jet transports ($M_{CR} \approx 0{,}8$) a simplified equation is given: Depending on the cruise altitude h_{CR} and by-pass ratio, BPR μ the thrust ratio is

$$\frac{T_{CR}}{T_{TO}} = (0{,}0013\,\mu - 0{,}0397)\frac{1}{\text{km}} h_{CR} - 0{,}0248\,\mu + 0{,}7125 \tag{5.28}$$

or with a cruise altitude in ft:

$$\frac{T_{CR}}{T_{TO}} = (3{,}962 \cdot 10^{-7}\,\mu - 1{,}210 \cdot 10^{-5})\frac{1}{\text{ft}} h_{CR} - 0{,}0248\,\mu + 0{,}7125 \tag{5.29}$$

5.6.2 Wing Loading

In cruise flight the lift is equal to the weight and the following applies:

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot q}{g} = \frac{C_L \cdot M^2}{g} \cdot \frac{q}{M^2} \quad . \tag{5.30}$$

q is the dynamic pressure calculated from $q = 1/2 \cdot \rho \cdot V^2$, M is the Mach number. The actual mass in cruise flight is less than m_{MTO} due to the fuel consumption since take-off. If we calculate with m_{MTO} here too, this again leads to a small safety margin with regard to the dimensioning.

Here the question arises as to what lift coefficient C_L is demanded in (5.30). The cruise phase must take place at an altitude where it is possible to reach the design lift coefficient specified for the profile. Often the design lift coefficient $C_{L,DESIGN} = C_{L,md}$ is chosen *for jets*. $C_{L,md}$ is the lift coefficient for minimum drag or for maximum lift-to-drag ratio. This lift coefficient is reached if the aircraft is flown at the speed of the lowest drag V_{md}. However, the speed V is practically already fixed due to the requirement for a cruise Mach number. We therefore choose a ratio V / V_{md} and therefore ultimately fix V_{md} and C_L. For a flight with maximum lift-to-drag ratio $V / V_{md} = 1.0$. A flight that produces the biggest range for a jet – and thus meets the range requirement most easily – requires $V / V_{md} = 1.316$ (see Flight Mechanics). If an aircraft has been optimized for slow flight, then its wing might be too big for cruise flight. The lift coefficient in cruise flight C_L is then less than $C_{L,md}$ and $V / V_{md} > 1.316$. However, the following should apply to many aircraft: $V / V_{md} = 1.0 \ldots 1.316$. Thus, by choosing V / V_{md}, the lift coefficient C_L in cruise flight is established, see equations (5.39) and (5.40). The practical significance of choosing V / V_{md} is that ultimately one has the option of moving the cruise flight curve in the matching chart (**Fig. 5.9**) and thus optimizing the design!

For q / M^2 in (5.25) the following is obtained:

$$\frac{q}{M^2} = \frac{\frac{1}{2} \cdot \rho \cdot V^2}{V^2 / a^2} = \frac{1}{2} \cdot \rho \cdot a^2 \quad . \tag{5.31}$$

We take the correlation for the sound velocity *a* from the thermodynamics

$$a^2 = \gamma \cdot \frac{p}{\rho} \tag{5.32}$$

γ is the ratio of specific heats (known as κ in the relevant German literature). For air, $\gamma = 1.4$ applies. When equation (5.32) is inserted in equation (5.31) this gives:

$$\frac{q}{M^2} = \frac{\gamma}{2} \cdot p(h) \quad . \tag{5.33}$$

The pressure $p(h)$ is determined from the standard atmosphere (see Flight Mechanics). Here it is important to bear in mind that the equation for the troposphere has to be used for an altitude *h* up to 11 km, and at an altitude *h* of between 11 km and 20 km the equation for the stratosphere applies.

Equation (5.33) inserted in equation (5.30) finally gives the wing loading as a function of the chosen parameters: lift coefficient C_L, Mach number M and altitude (h).

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot M^2}{g} \cdot \frac{\gamma}{2} \cdot p(h) \quad . \tag{5.34}$$

The results of a separate calculation of **wing loading and** the **thrust-to-weight ratio** are entered at the end of the cruise analysis in a table like **Table 5.4**.

Table 5.4 Example table for the collection of cruise performance data

altitude h	wing loading m/S	thrust-to-weight ratio $T/(m \cdot g)$
...	...	...
…	…	…
5000 m	...	...
...	...	...
…	…	…

The values can then be transferred from this table to the matching chart, thus producing the function $T/(m \cdot g) = f(m/S)$.

5.7 Lift-to-Drag Ratio during Cruise

Lift-to-drag ratio not only increases with increasing wing aspect ratio, but also with a small wetted area of the aircraft relative to the wing area S_{wet} / S_W. **Fig. 5.7** shows that the lift-to-drag ratio is a function of the

$$\text{"Wetted Aspect Ratio"} = A / \left(S_{wet} / S_W \right) . \tag{5.35}$$

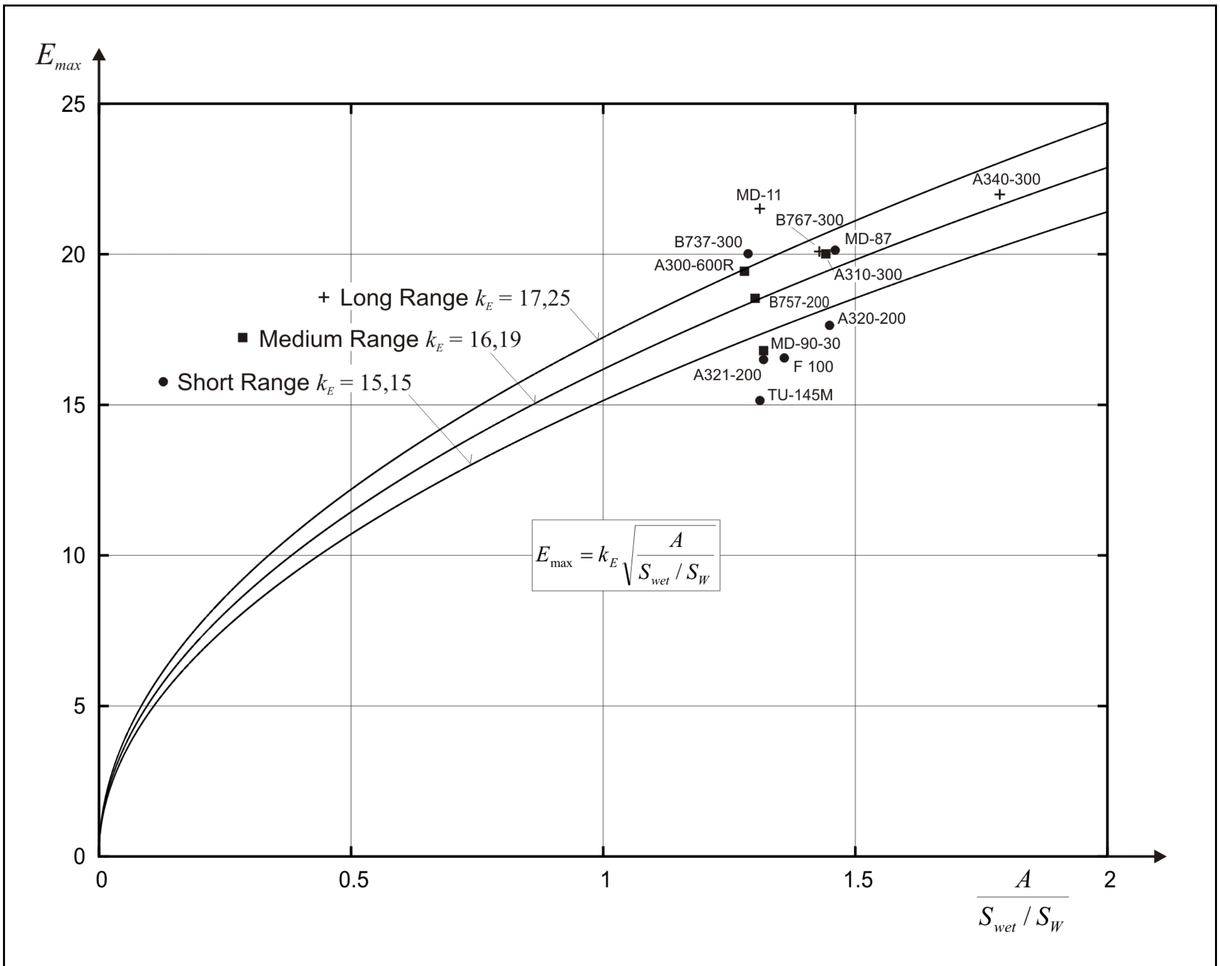


Fig. 5.7 Estimation of glide ratio, wetted area and wing area (based on **Raymer 1989**)

The relationship of **Fig. 5.7** can also be expressed by equations. Takings a closer look at underlying principles, it becomes apparent that one is dealing with functions $y = \sqrt{x}$ in **Fig. 5.7**. This can be derived, but it will be dispensed with at this point. It is then

$$E_{max} = k_E \sqrt{\frac{A}{S_{wet} / S_W}} \quad . \tag{5.36}$$

A derivation would yield

$$k_E = \frac{1}{2}\sqrt{\frac{\pi\, e}{\overline{c_f}}} \tag{5.37}$$

Loftin 1980 chooses $e = 0.85$ for all jet aircraft in the cruise configuration. $\overline{c_f} = 0.003$ is a common value in literature for jet transports. Thus, giving

$$k_E = \frac{1}{2}\sqrt{\frac{\pi\, e}{\overline{c_f}}} = 14.9 \quad .$$

k_E, according to the data used by **Raymer 1989** (**Fig. 5.7** evaluated) gives

$$k_E = 15.8 \quad .$$

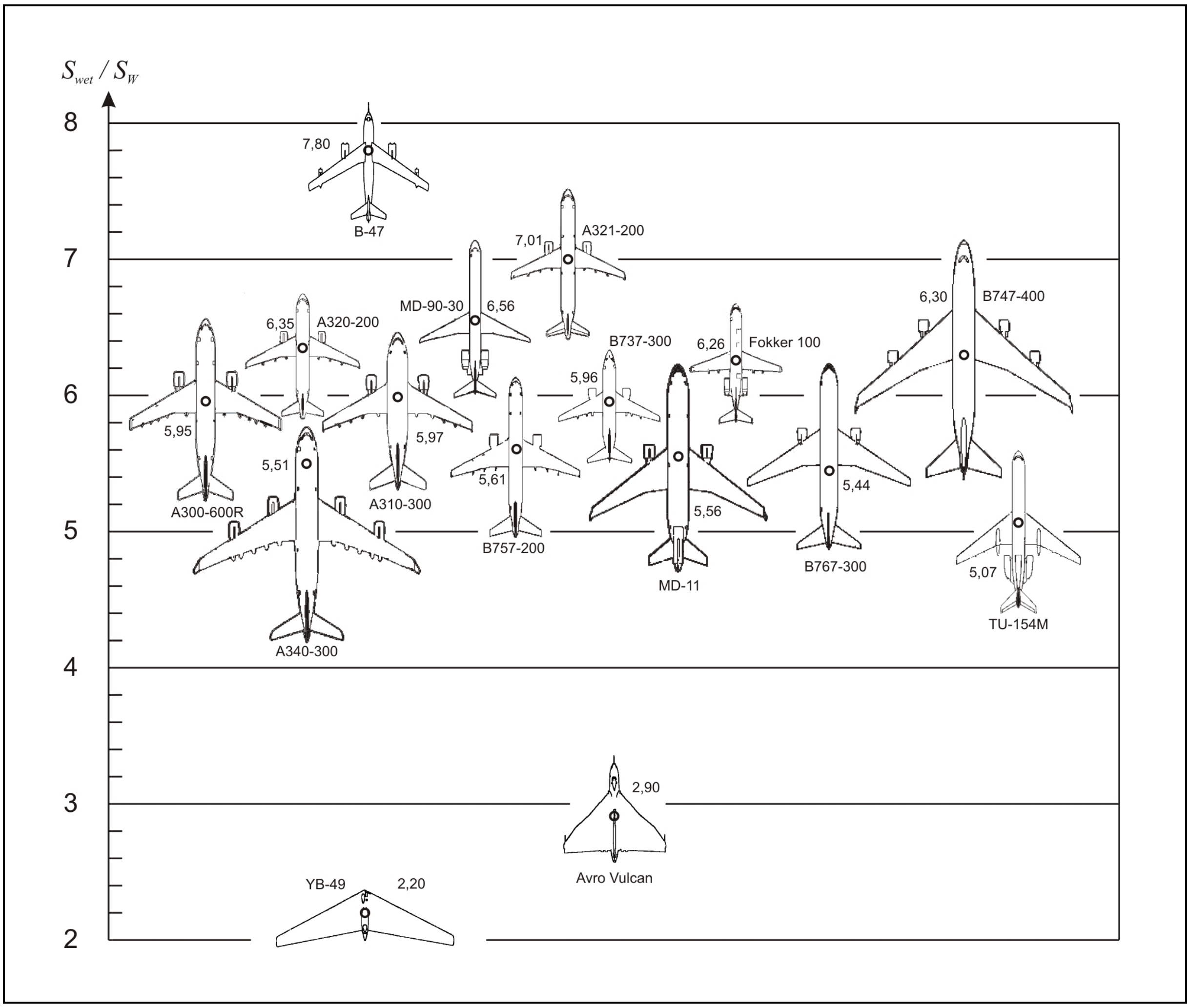


Fig. 5.8 Aircraft plan forms and their relative wetted area S_{wet} / S_W (based on **Raymer 1989**)

Fig. 5.8 illustrates which aircraft categories have which ratio S_{wet} / S_W and shows for conventional aircraft configurations:

$$S_{wet} / S_W = 6.0 \ldots 6.2 \tag{5.38}$$

Lift coefficient in cruise flight for flight with minimum drag, i.e. with E_{max} :

$$\boxed{C_{L,md} = \frac{\pi\, A\, e}{2 E_{max}}} \tag{5.39}$$

Actual lift coefficient devided by lift coefficient for flight with minimum drag

$$C_L / C_{L,md} = 1/(V / V_{md})^2 \quad \text{and therefore}$$

$$\boxed{C_L = \frac{C_{L,md}}{(V / V_{md})^2}} \tag{5.40}$$

The actual lift-to-drag ratio in cruise flight is

$$\boxed{E = \frac{2\, E_{max}}{\dfrac{1}{\left(\dfrac{C_L}{C_{L,md}}\right)} + \left(\dfrac{C_L}{C_{L,md}}\right)}} \tag{5.41}$$

5.8 Matching Chart

In the matching chart a two-dimensional optimization problem is solved graphically. The two **optimization variables** are:

- thrust-to-weight ratio, $T_{TO} / (m_{TO} \cdot g)$ and
- wing loading, m_{MTO} / S_W .

In previous sections it was demonstrated how, from the various performance requirements, either the wing loading or the thrust-to-weight ratio can be calculated. For all calculations it was ensured that wing loading and thrust-to-weight ratio always refer to take-off with MTOW, which made it possible to compare the values of different flight phases. The results are plotted on the matching chart. **Fig. 5.9** shows such a hypothetical matching chart.

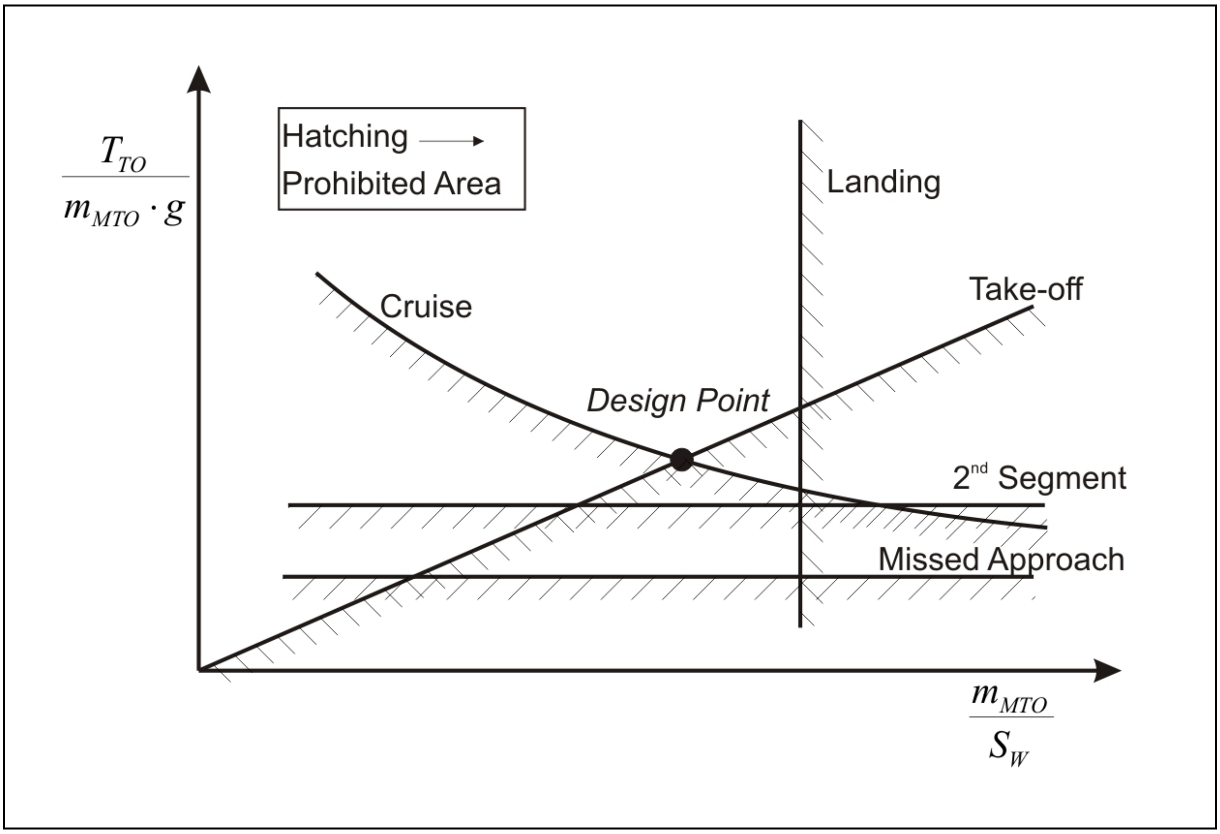


Fig. 5.9 Hypothetical matching chart

The aim of optimization is to achieve the following:
- *Priority 1*: to achieve the smallest possible thrust-to-weight ratio;
- *Priority 2*: to achieve the highest possible wing loading.

The resultant pair of values with the elements "wing loading" and "thrust-to-weight ratio" constitutes a solution to the design problem which meets the examined constraints and also involves a comparatively low weight.

The results thus gained should still be examined for plausibility. To do this, statistical values of designed aircraft can be referred to, as contained in **Fig. 5.10** and **Fig. 5.11**, as well as in **Table 5.5** and **Table 5.6**.

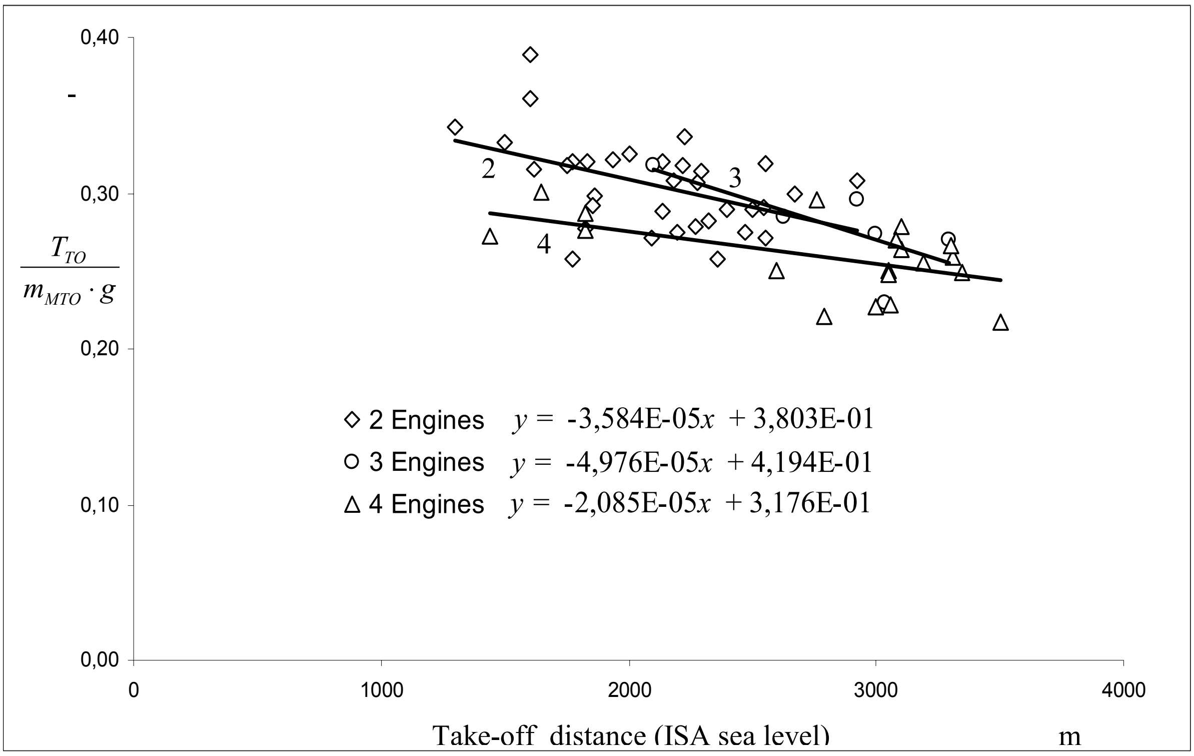


Fig. 5.10 Thrust-to-weight ratio as a function of balanced field length (data from **Jenkinson 1999**)

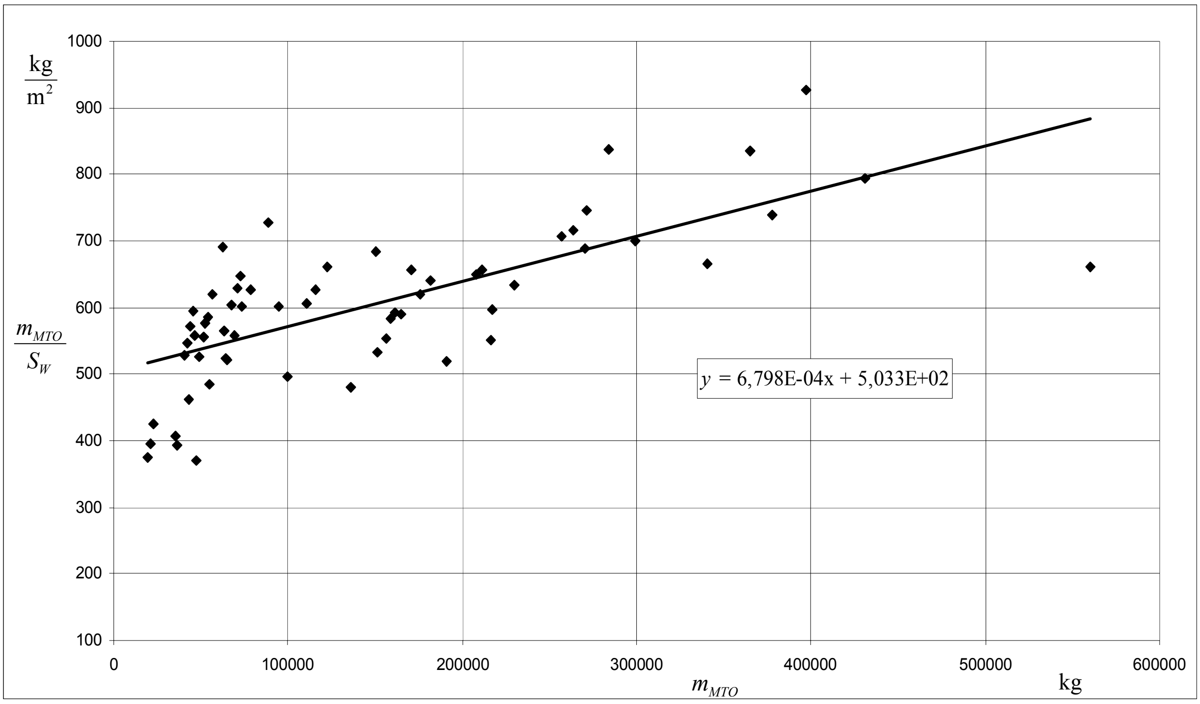


Fig. 5.11 Wing loading as a function of maximum take-off mass (data from **Jenkinson 1999**)

Table 5.5 Thrust- respectively power-to-weight ratio of different types of aircraft (based on **Raymer 1989)**

type of aircraft	typical value	unit	
jet transport	0.25	-	$T_{TO} / m_{MTO} \cdot g$
single engine piston propeller	12	W/N	$P_{TO} / m_{MTO} \cdot g$
twin engine piston propeller	28	W/N	$P_{TO} / m_{MTO} \cdot g$
twin turboprop	34	W/N	$P_{TO} / m_{MTO} \cdot g$

Table 5.6 Wing loading of different types of aircraft (based on **Raymer 1989**)

type of aircraft	m_{MTO} / S_W (kg/m²)
glider	29
homebuilt	54
single engine piston propeller	83
twin engine piston propeller	127
twin turboprop	195
jet transport	586

5.9 Maximum Take-Off Mass

The maximum take-off mass m_{MTO} is comprised of payload, fuel mass (for a specific range R at a specific payload m_{PL}) and the operating empty mass:

$$m_{MTO} = m_{PL} + m_F + m_{OE} \quad . \tag{5.42}$$

If we recast this, we get

$$m_{MTO} - m_F - m_{OE} = m_{PL} \tag{5.43}$$

$$m_{MTO} \cdot \left(1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}} \right) = m_{PL} \tag{5.44}$$

$$\boxed{m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \quad .} \tag{5.45}$$

The relative fuel mass m_F / m_{MTO} and relative operating empty mass m_{OE} / m_{MTO} are discussed in two sub-sub sections that follow.

5.9.1 Relative Operating Empty Mass

The relative operating empty mass m_{OE} / m_{MTO} or relative useful load u are estimated from aircraft statistics. Definitions are

$$u = \frac{m_F + m_{PL}}{m_{MTO}} = 1 - \frac{m_{OE}}{m_{MTO}} \quad . \tag{5.46}$$

Two approaches are given here to calculate m_{OE} / m_{MTO}.

<u>**Approach 1**</u>**:**
Marckwardt 1998a uses a regression calculation for *jet transports*:

$$\boxed{\frac{m_{OE}}{m_{MTO}} = 0.591 \cdot \left(\frac{R\,[\mathrm{km}]}{1000} \right)^{-0.113} \cdot \left(\frac{m_{MTO}\,[\mathrm{kg}]}{1000} \right)^{0.0572} \cdot n_E^{-0.206} \quad .} \tag{5.47}$$

Equation (5.47) provided m_{OE} / m_{MTO} for all aircraft examined by **Marckwardt 1998a** with an error rate of less than 10%. Note: equation (5.47) has to be used iteratively:

1. select a starting value m_{OE} / m_{MTO} = 0.5
2. insert m_{OE} / m_{MTO} into equation (5.45) and obtain (with m_F / m_{MTO} from 5.9.2) m_{MTO}
3. calculate a new value for m_{OE} / m_{MTO} from equation (5.47)
4. go back to step 2 and repeat until convergence.

Approach 2:
Loftin 1980 (unlike other authors) uses the thrust-to-weight ratio obtained in the preliminary sizing procedure to determine the relative operating empty mass or the relative useful load *u* from a statistical analysis. Various civil jets from a business jet to a Boeing 747 were included in the analysis, and a thrust-to-weight ratios of between 0.23 and 0.46 was taken into account. The result can be summarized (**Loftin 1980,** Fig. 3.21)

$$u = 0.77 - 1.04 \cdot \frac{T_{TO}}{m_{MTO} \cdot g} \tag{5.48}$$

or

$$\boxed{\frac{m_{OE}}{m_{MTO}} = 0.23 + 1.04 \cdot \frac{T_{TO}}{m_{MTO} \cdot g} \quad .} \tag{5.49}$$

Equation (5.49) provided the relative operating empty mass m_{OE} / m_{MTO} for virtually all aircraft examined by **Loftin 1980** with an error rate of less than 10%. The relative operating empty mass m_{OE} / m_{MTO} increases with increasing thrust-to-weight ratio. As a high thrust-to-weight ratio requires high-performance and therefore heavy engines, equation (5.49) reflects the expected tendency. Furthermore (5.49) is in very good agreement with independent statistical data from **Fig. 5.12**.

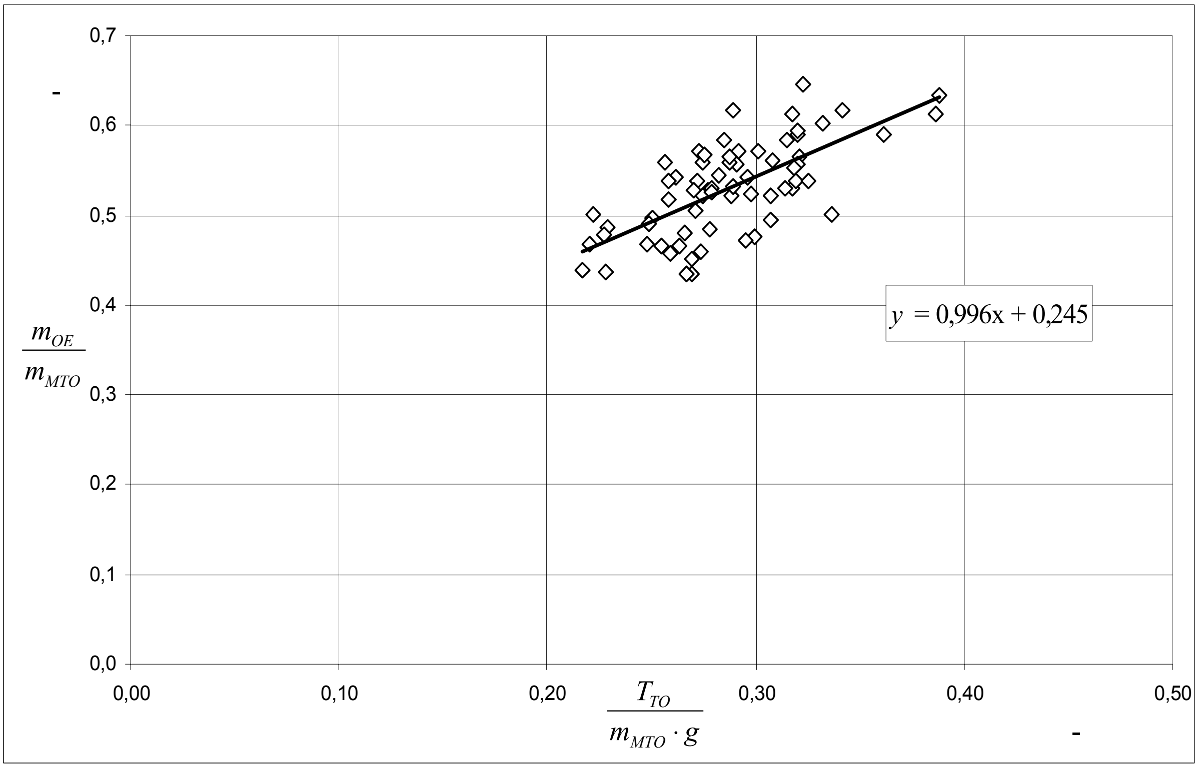


Fig. 5.12 Relative operating empty mass m_{OE} / m_{MTO} as a function of thrust-to-weight ratio (data from **Kallmeyer 1999**, **Jenkinson 1999**)

Fig. 5.13 and **Fig. 5.14** give further inside into dependencies of m_{OE} / m_{MTO} .

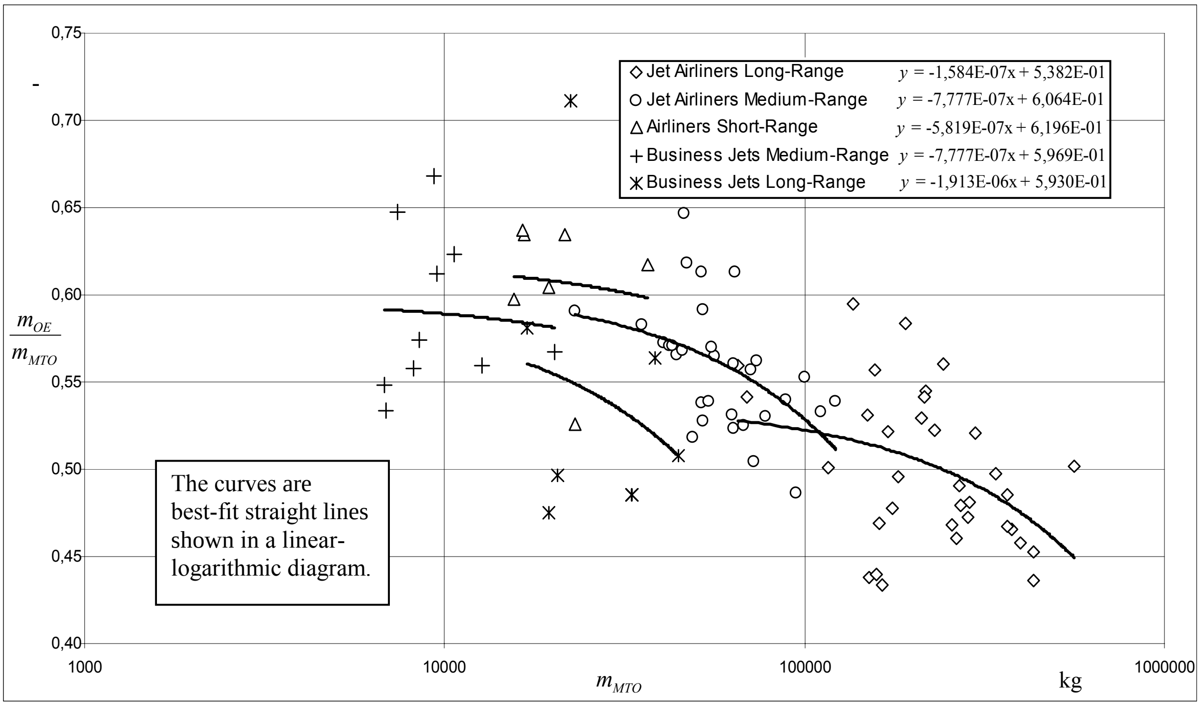


Fig. 5.13 Relative operating empty mass m_{OE} / m_{MTO} as a function of maximum take-off mass m_{MTO} (data from **Jenkinson 1999**, www.wikipedia.de)

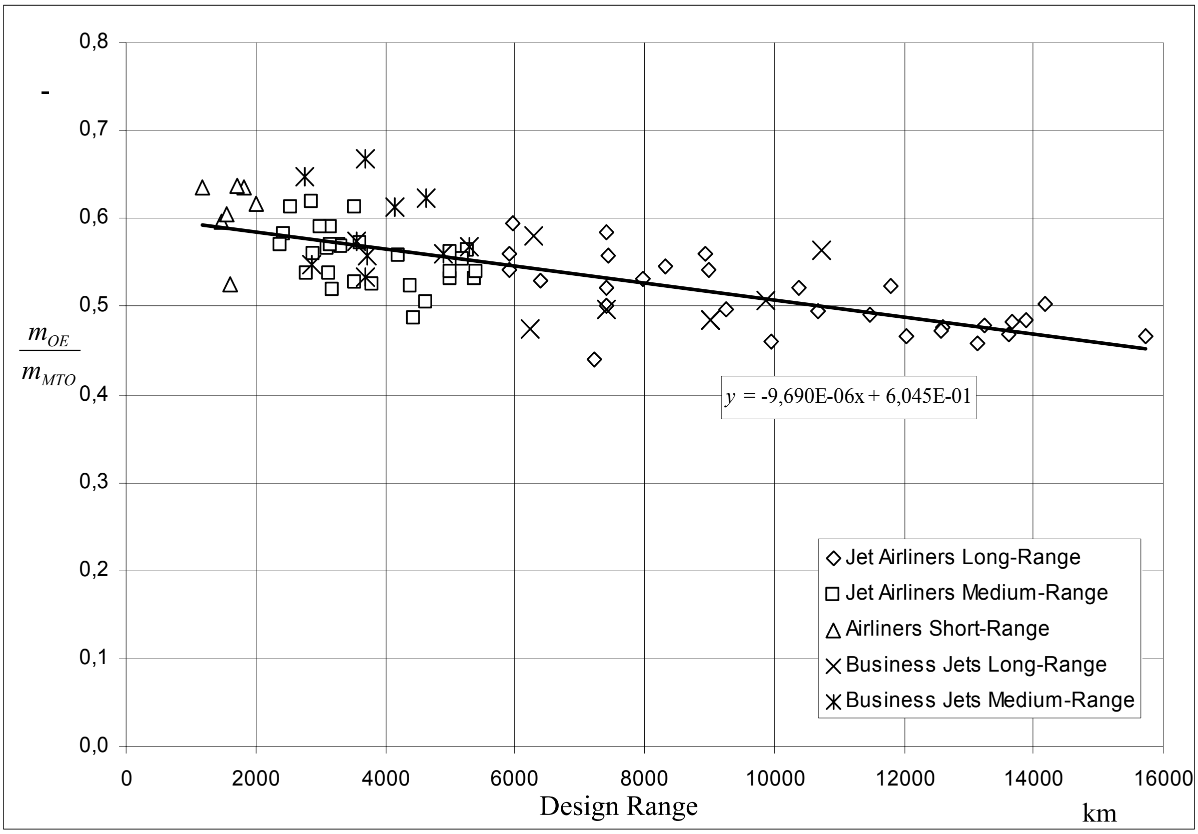


Fig. 5.14 Relative operating empty mass m_{OE} / m_{MTO} as a function of design range (no fuel reserves) (data from **Jenkinson 1999** and www.wikipedia.de**)**

5.9.2 Relative Fuel Mass

The relative fuel mass m_F / m_{MTO} is inserted in equation (5.45) to estimate the maximum take-off mass m_{MTO}. Fuel is required during all flight phases from starting the engines to taxiing off after landing. The flight phases can be named as shown in **Fig. 5.15**. To simplify the calculation the descent (DES) is often omitted. Instead it can be assumed that the distance covered during descent is already covered during cruise flight.

m_i is the mass at the beginning of a flight phase (i = *TO*, *CLB*, *CR*, ...). m_{i+1} is the mass at the start of the next flight phase. m_L is the mass at the beginning of the landing phase, m_T mass at the beginning of "taxi to apron". Lets call m_{SO} the mass at the end the flight "after switch off". The parameter m_{i+1} / m_i refers to flight phase i and is called *mission segment mass fraction*. The parameter $1 - m_{i+1} / m_i$ is then the relative fuel consumption in the respective flight phase i. The flight phases engine start (*ES*) and taxi (*T*) can be omitted if only the take-off mass has to be calculated. This is the case here. All *mission segment mass fractions* taken together then provide a parameter for calculating the fuel consumption for the entire flight: This parameter is called *mission fuel fraction* M_{ff} .

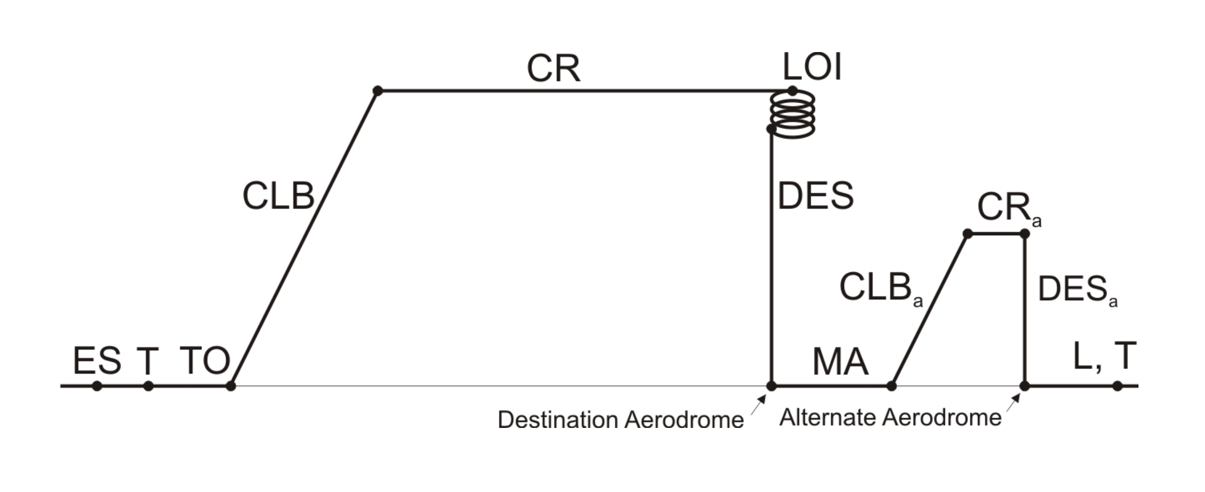


Fig. 5.15 Typical flight phases of a civil transport flight mission

$$M_{ff} = \frac{m_{SO}}{m_T} \cdot \frac{m_T}{m_L} \cdot \frac{m_L}{m_{DES}} \cdot \frac{m_{DES}}{m_{CR,alt}} \cdot \frac{m_{CR,alt}}{m_{CLB}} \cdot \frac{m_{CLB}}{m_{MA}} \cdot \frac{m_{MA}}{m_{DES}} \cdot \frac{m_{DES}}{m_{LOI}} \cdot \frac{m_{LOI}}{m_{CR}} \cdot \frac{m_{CR}}{m_{CLB}} \cdot \frac{m_{CLB}}{m_{TO}} = \frac{m_{SO}}{m_{TO}} \tag{5.50}$$

The entire mass of the fuel consumed on the flight is then calculated from the *mission fuel fraction* M_{ff}

$$m_F = m_{TO} - m_{SO} = m_{TO} \cdot \frac{m_{TO} - m_{SO}}{m_{TO}} = m_{TO} \cdot \left(1 - M_{ff}\right) \ . \tag{5.51}$$

The relative fuel mass for equation (5.45) follows from the mission fuel fraction

$$\frac{m_F}{m_{TO}} = 1 - M_{ff} \ . \tag{5.52}$$

The *mission segment mass fractions* m_{i+1} / m_i first have to be determined in order to be able to work with equation (5.50) and (5.52):

- The mass ratios for cruise and loiter *must* be determined according to *Breguet* (see below).
- For the remaining flight phases it is scarcely possible or worthwhile to calculate the mass ratio with the resources available here, so that the data in **Table 5.9** has to be resorted to.

For the cruise flight of a jet, the Breguet range factor is

$$B_s = \frac{L/D \cdot V}{SFC_T \cdot g} \quad . \tag{5.53}$$

For the cruise flight of a propeller aircraft the corresponding Breguet range factor is

$$B_s = \frac{L/D \cdot \eta}{SFC_P \cdot g} \quad . \tag{5.54}$$

In equation (5.54) $c = SFC_T$ is the *thrust*-specific fuel consumption. In equation (5.55) SFC_P is the *performance*-specific fuel consumption and η is the propeller efficiency. The mission segment mass fraction for the cruise phase then comes to the following with the Breguet range factor B_s

$$\frac{m_{LOI}}{m_{CR}} = e^{-\frac{s_{CR}}{B_s}} \quad . \tag{5.55}$$

s_{CR} is the distance covered in the cruise phase. **Table 5.7** and **Table 5.8** provide information on the specific fuel consumption.

More details to the calculation of fuel mass (taking into account the regulations on fuel reserves) are given in a spreadsheet based method for aircraft preliminary sizing that accompanies these lecture notes.

Table 5.7 Specific fuel consumption $c = SFC_T$ for jets (based on **Raymer 1989**)

SFC_T	cruise lb/lb/h	mg/N/s	loiter lb/lb/h	mg/N/s
turbojet	0.9	25.5	0.8	22.7
turbofan, low bypass ratio	0.8	22.7	0.7	19.8
turbofan, high bypass ratio	0.5	14.2	0.4	11.3

Table 5.8 Specific fuel consumption SFC_P and propeller efficiency η for propeller aircraft (based on **Raymer 1989**)

	cruise SFC_P		η	loiter SFC_P		η
	lb/hp/h	mg/W/s	-	lb/hp/h	mg/W/s	-
piston, fixed pitch propeller	0.4	0.068	0.8	0.5	0.085	0.7
piston, variable pitch propeller	0.4	0.068	0.8	0.5	0.085	0.8
turboprop	0.5	0.085	0.8	0.6	0.101	0.8

Table 5.9 Generic mission segment mass fractions (based on **Roskam I**)

type of aircraft	engine start	taxi	take-off	climb	descent	landing
business jet	0.99	0.995	0.995	0.98	0.99	0.992
jet transport	0.99	0.99	0.995	0.98	0.99	0.992
fighter	0.99	0.99	0.99	0.96 – 0.9	0.99	0.995
supersonic cruise	0.99	0.995	0.995	0.92 – 0.87	0.985	0.992

5.10 Take-off Thrust and Wing Area

Take-off thrust and wing area can easily be calculated with the now known maximum take-off mass m_{MTO} from the thrust-to-weight ratio $T_{TO} / \left(m_{MTO} \cdot g \right)$ and the wing loading m_{MTO} / S_W

$$T_{TO} = m_{MTO} \cdot g \cdot \left(\frac{T_{TO}}{m_{MTO} \cdot g} \right) \tag{5.56}$$

$$S_W = m_{MTO} / \left(\frac{m_{MTO}}{S_W} \right) \quad . \tag{5.57}$$

Landing mass m_L, operating empty mass m_{OE}, fuel mass m_F and some other parameters can now easily be calculated.