The Conflict of Aerodynamic Efficiency and Static Longitudinal Stability of Box Wing Aircraft

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Abstract

The induced drag of box wing aircraft is assessed with the help of literature data. The theoretical foundations of static longitudinal stability and controllability are presented and applied to the box wing aircraft. The results are interpreted and put into practice with the help of a medium range box wing aircraft based on the Airbus A320. Stability in cruise is attained by increasing the ratio $C_{L,1}/C_{L,2}$ to a value of 1,74, which is the ratio of lift coefficients of the forward and the aft wing. According to biplane theory this results in a 3,4 % increase of induced drag. Applying aerodynamic theory for closed wing systems no increase would be expected. With the stated ratio of lift coefficients results a relatively small envelope for the center of gravity (CG). Consequently the aircraft is designed to be well balanced with regard to its CG. The individual CGs of the airframe, engines, fuel and payload are all located approximately at the same position. Hence the CG shift is minimized for different payload and fuel quantities.

1 Introduction

As stated in Flightpath 2050 civil aviation transport is facing challenges like globalization, climate change and a cumulative scarcity of resources [1]. To cope with these challenges, aircraft have to become more efficient, especially concerning energy and fuel consumption. Flightpath 2050 states: "In 2050 technologies and procedures available allow a 75% reduction in CO2 emissions per passenger kilometre ... these are relative to the capabilities of typical new aircraft in 2000." Benefits of alternative fuels are seen in addition to this. [1]

It is questionable, if these goals are realistic. Without doubt the latest aircraft emerging on the market have pretty much exhausted the inherent saving potential of conventional configurations without reaching the goal of a 75% CO2 (or fuel) reduction. Therefore it becomes a necessity also to exploit all saving potentials originating from the aircraft configuration. It is not a question anymore if unconventional configurations find acceptance or not. Quite simply, without unconventional configurations stated Flightpath 2050 goals will not be reached. One of these configurations is the box wing aircraft, a biplane with oppositely swept wings whose tips are connected by extended winglets (see Fig. 1). The most recognized benefits of this configuration are its low induced drag and alleged structural superiority.

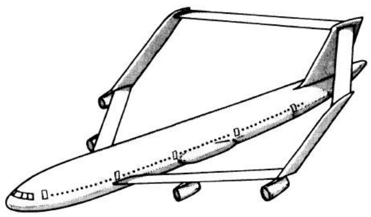

Fig. 1 Example of a box wing aircraft [2]

From literature it can be understood that it is difficult to attain static longitudinal stability and controllability for box wing aircraft. Civil transport aircraft have to be stable in flight [3]. Consequently the design of the aircraft is greatly influenced by the requirements according to static longitudinal stability and controllability. In this paper these requirements are analyzed and possibilities for attaining stability are presented. The easiest to accomplish is having different lift coefficients at both wings which, in the case of equal wing areas, leads to an unequal division of lift between both wings. Applying biplane theory this results in an increase of induced drag [4]. Thus assuring stability is in conflict with aerodynamic efficiency.

In this paper the above approach is applied to the design of a medium range box wing aircraft. Its special design characteristics because of stability requirements are outlined and the decrease of aerodynamic efficiency according to biplane theory is assessed.

2 Aerodynamic Efficiency of Box Wing Aircraft

2.1 Induced Drag

A prominent aerodynamic feature of the box wing configuration is its low induced drag $(D_i)_{box}$. It can be expressed in relation to the induced drag $(D_i)_{ref}$ of a conventional reference wing with the same span. The published equations mostly have the form

$$\frac{(D_i)_{box}}{(D_i)_{ref}} = \frac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b} \qquad (1)$$

where h/b is the height to span ratio, which is the vertical gap between both wings divided by the wing span of the whole configuration. k_1 up to k_4 are free parameters which are adjusted so that the result of Eq. (1) comes close to measured or calculated data. Equation (1) is only applicable when both the box wing and the reference wing configuration have their optimum span loading. In literature there exist several solutions for the parameters k_1 to k_4. A common result is given in [4] reading

$$\frac{(D_i)_{box}}{(D_i)_{ref}} = \frac{1 + 0{,}45 \cdot h/b}{1{,}04 + 2{,}81 \cdot h/b} \qquad (2)$$

which slightly overestimates the actual reduction of induced drag, as confirmed in [5]. For the design of the medium range box wing aircraft a more conservative approach is taken, coming from [6]:

$$\frac{(D_i)_{box}}{(D_i)_{ref}} = \frac{0{,}44 + 0{,}9594 \cdot h/b}{0{,}44 + 2{,}219 \cdot h/b} \qquad (3)$$

Figure 2 shows a graphic comparison of the Eqs. (2) and (3).

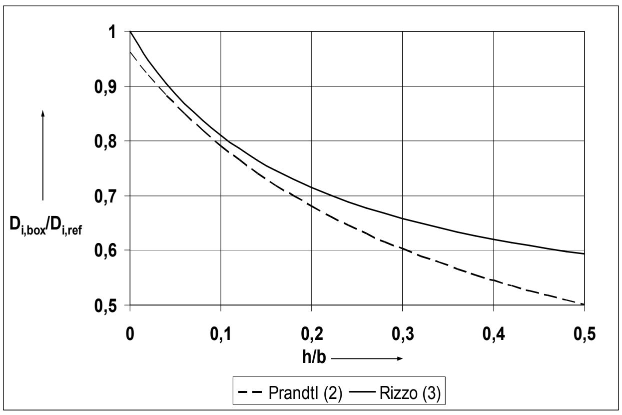

Fig. 2 Reduction of induced drag acc. to Eqs. (2) and (3)

2.2 Conditions for Minimum Induced Drag

For a conventional cantilever wing the lift distribution for minimum induced drag is elliptical. The conditions concerning the box wing configuration are more complex, since the span wise distribution and also the division of lift between both wings have to be considered.

2.2.1 Span Wise Lift Distribution

In [7] it is indicated that the lift distribution for minimum induced drag is a combination of an elliptical and a constant part for the horizontal wings and a linear and butterfly shaped part for the vertical winglets, as it is shown in Fig. 3.

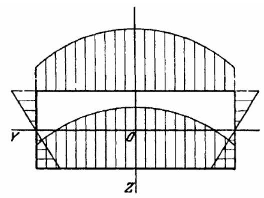

Fig. 3 Lift distribution of a box wing configuration [7]

As depicted in [8] the ratio of the constant to the elliptical part increases with higher h/b ratios. The investigations in [8] were performed for cantilever wing-winglet configurations, but the qualitative results should be transferable to box wing configurations.

2.2.2 Lift Division between Both Wings

According to biplane theory both wings have to generate the same amount of lift so that minimum induced drag is achieved [4]. With the help of Eq. (4) presented in [4] the induced drag of an arbitrary biplane can be calculated. It reads

$$(D_\mathrm{i})_\mathrm{bi} = \frac{1}{\pi q}\left(\frac{L_1{}^2}{b_1{}^2} + 2\sigma\frac{L_1 L_2}{b_1 b_2} + \frac{L_2{}^2}{b_2{}^2}\right) \qquad (4)$$

where q is the dynamic pressure, L_i the lift of the individual wings, b_i the span of the individual wings and σ is the wing interference factor which depends on the h/b ratio. In [4] the equation for calculating the minimum induced drag of a biplane is derived as well, being

$$(D_\mathrm{i})_\mathrm{bi,min} = \frac{L^2}{\pi q b^2}\cdot\frac{1+\sigma}{2} \qquad (5)$$

where L is the total lift and b the span of both wings. Equation (5) is only applicable if both wings generate $L/2$ and have equal wing spans.

Now the relation of the actual drag of a biplane with equal wing spans to the minimum induced drag can be formulated. It is assumed that the result is applicable to box wing configurations as well. Dividing Eq. (4) by Eq. (5) and introducing the lift ratio

$$x = \frac{L_1}{L_2} \qquad (6)$$

yields

$$\frac{D_\mathrm{i}}{D_\mathrm{i,min}} = 2\frac{x^2 + 2\sigma x + 1}{(x+1)^2(\sigma+1)}. \qquad (7)$$

Using

$$\sigma = 2\frac{(D_\mathrm{i})_\mathrm{box}}{(D_\mathrm{i})_\mathrm{ref}} - 1 \qquad (8)$$

which is concluded from [5] and taking Eq. (3) it is possible to plot the ratio of the actual to the minimum induced drag depending on the h/b ratio and the lift ratio (Fig. 4).

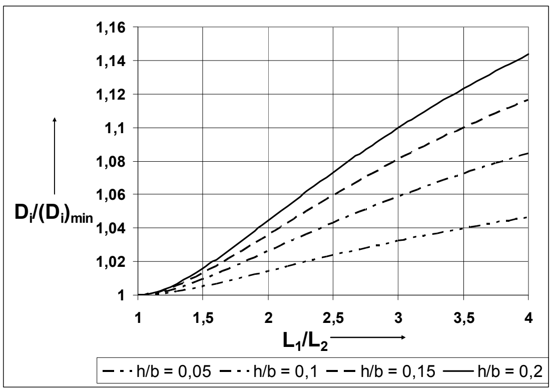

Fig. 4 Induced drag penalties according to biplane theory

The derivation of Eq. (7) can be found in [9]. In section 4.2 it will be shown that the lift ratio has a value about two. For h/b ratios of about 0,2, which is a reasonable value for actual aircraft, a 4 % increase of induced drag would occur.

The induced drag penalty results when applying biplane theory. But there also exists a different approach which takes account of the characteristics of a closed wing system. Amongst others it is presented in [10] and [11] and states that for a closed wing system minimum induced drag can also be achieved when both horizontal wings generate a different amount of lift. This is possible by adding a constant circulation loop to the distribution of circulation of the whole wing configuration. An example of the resulting lift distribution is depicted in Fig. 5. Adding a constant circulation loop does not change the elliptical parts, but the constant part of one horizontal wing is reduced while that of the other wing is increased by the same amount. Consequently the winglet loading changes as well.

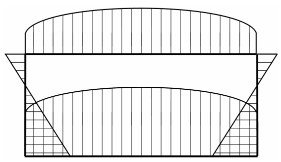

Fig. 5 Lift distribution with unequal lift of both wings

[2] and [5] apply the conditions according to biplane theory to the design of a box wing aircraft. This approach was taken for the medium range box wing aircraft presented in section 4 of this paper. The applicability of the proposals in [10] and [11] is part of ongoing studies.

3 Static Longitudinal Stability and Controllability of Box Wing Aircraft

3.1 Basics

For assessing static longitudinal stability and controllability of a box wing aircraft its CG envelope is taken as reference. It is the permissible region of the aircraft's center of gravity. The forward limit is defined by controllability requirements (control limit). The aft limit is the neutral point, which is defined by stability requirements (stability limit). For a stable and controllable aircraft the control limit has to be situated in front of the stability limit.

The condition for a controllable aircraft is a positive pitching moment about the center of gravity (M_{CG}) which counters the negative pitching moment generated by the wings. With the help of the pitching moment coefficient $C_{M,CG}$ this condition reads

$$C_{M,\mathrm{CG}} > 0\,. \tag{9}$$

There are two conditions for a stable aircraft. The stability condition requires that the slope of the pitching moment coefficient with regard to the lift coefficient C_L is negative [12]:

$$\frac{\mathrm{d}C_{M,\mathrm{CG}}}{\mathrm{d}C_L} < 0\,. \tag{10}$$

The trim condition requires that the pitching moment coefficient at zero lift is positive [12]:

$$\left(C_{M,\mathrm{CG}}\right)_{C_L=0} > 0\,. \tag{11}$$

The conditions given with the Ineqs. (9), (10) and (11) are applied to the equilibrium of moments of the aircraft in horizontal and non-accelerated flight neglecting control surfaces or a horizontal stabilizer. This results in a formulation of the stability limit and the control limit of the CG.

3.2 Equilibrium of Moments in Horizontal and Non-Accelerated Flight Neglecting Control Surfaces

The formation of the equilibrium of moments is based on the scheme shown in Fig. 6.

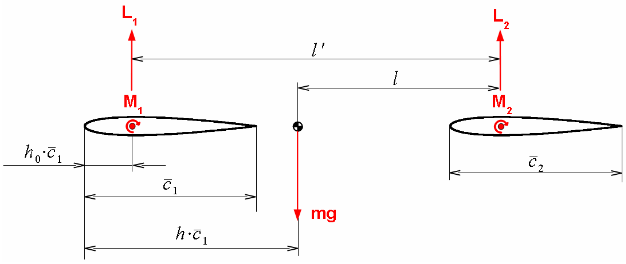

Fig. 6 Forces and moments acting on a box wing aircraft

The lift forces L_1 and L_2, the weight mg and the pitching moment of the wings about their aerodynamic centers M_1 and M_2 are considered. Drag and thrust are neglected, so that the vertical distance between both wings is irrelevant. The CG position h is expressed as multiple of the length of the mean aerodynamic chord (MAC) of the forward wing $\bar{c}_1$, measured from the leading edge of MAC$_1$. The position of the aerodynamic center of the forward wing h_0 is also expressed as multiple of $\bar{c}_1$, measured from the leading edge of MAC$_1$. For simplification h_0 is assumed to be 0,25. The distance between the aerodynamic centers of both wings is given with l', the distance between the CG and the aerodynamic center of the aft wing with l. $\bar{c}_2$ is the length of MAC$_2$. The aerodynamic centers are assumed to be at 25 % of the respective MAC.

The following equations only show results of the formulation of the equilibrium of moments. A detailed derivation is presented in [9]. According to Fig. 6 the equilibrium of moments reads

$$M_{CG} = L_1 (h - h_0)\bar{c}_1 - L_2 l + M_1 + M_2 = 0. \qquad (12)$$

Introducing the total lift L and building the coefficient form of Eq. (12) results in

$$C_{M,CG} = C_L (h - h_0)\bar{c}_1' - C_{L,2}\bar{V}' + C_{M,1}\bar{c}_1' s_1 + C_{M,2}\bar{c}_2' s_2 = 0 \qquad (13)$$

with $\bar{c}_i'$ as the relative length of the respective MAC given by

$$\bar{c}_i' = \frac{\bar{c}_i}{\bar{c}}; \quad i = 1;2 \qquad (14)$$

where the length of the total MAC $\bar{c}$ is defined with

$$\bar{c} = \bar{c}_1 s_1 + \bar{c}_2 s_2 \qquad (15)$$

according to [9]. s_1 and s_2 are the relative reference areas of the individual wings:

$$s_i = \frac{S_i}{S_1 + S_2}; \quad i = 1;2. \qquad (16)$$

$\bar{V}'$ is the modified volume coefficient of the aft wing. It is defined with

$$\bar{V}' = \frac{l'}{\bar{c}} s_2. \qquad (17)$$

3.3 CG Envelope

Applying the conditions given by the Ineqs. (9), (10) and (11) to Eq. (13) results in a formulation of the control and the stability limit of the CG position. The condition for controllability reads

$$h > h_0 + \frac{C_{L,2}}{C_L} \cdot \frac{\bar{V}'}{\bar{c}_1'} + \frac{C_{M,1}}{C_L} s_1 + \frac{C_{M,2}}{C_L} s_2 \frac{\bar{c}_2}{\bar{c}_1}. \qquad (18)$$

Here all parameters are either defined by aircraft geometry or the flight state. The condition for stability is

$$h < h_0 + \frac{dC_{L,2}}{dC_L} \cdot \frac{\bar{V}'}{\bar{c}_1'} \qquad (19)$$

where the determination of the gradient $dC_{L,2}/dC_L$ requires special attention (see section

3.4). The other parameters are defined by aircraft geometry. The trim condition reads

$$C_{M,1}c_1's_1 + C_{M,2}c_2's_2 - (C_{L,2})_{C_L=0} \cdot \bar{V}' > 0 . \quad (20)$$

The lift coefficient of the aft wing at zero total lift $(C_{L,2})_{C,L=0}$ can be determined with the help of the gradient $\mathrm{d}C_{L,2}/\mathrm{d}C_L$. The appropriate equation is

$$(C_{L,2})_{C_L=0} = C_{L,2} - \frac{\mathrm{d}C_{L,2}}{\mathrm{d}C_L} \cdot C_L , \quad (21)$$

assuming that the gradient $\mathrm{d}C_{L,2}/\mathrm{d}C_L$ is constant. All other parameters in Ineq. (20) are geometric or aerodynamic constants. The pitching moment $C_{M,\mathrm{i}}$ of all airfoils is assumed to be -0,1 which is sufficient for the preliminary design of the medium range box wing aircraft.

3.4 Determination of $\mathrm{d}C_{L,2}/\mathrm{d}C_L$

[12] shows that it is possible to calculate the gradient $\mathrm{d}C_{L,2}/\mathrm{d}C_L$ with the help of the following expression:

$$\frac{\mathrm{d}C_{L,2}}{\mathrm{d}C_L} = \frac{a_2}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) \quad (22)$$

where a_2 is the lift curve slope of the isolated aft wing and a the lift curve slope of the whole aircraft. ε is the downwash angle and α the angle of attack with regard to the zero lift line of the aircraft. a_2 is estimated with the help of Eq. (23) given in [13].

$$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = \frac{2\pi \cdot A}{2 + \sqrt{A^2\left(1 + \tan^2 \varphi_{50} - M^2\right) + 4}} . \quad (23)$$

A is the wing aspect ratio, φ_{50} the sweep angle at half chord and M the Mach number. The total lift curve slope a is determined according to section 4.5.1.1 in [14]. There the influence on the aft wing because of the vorticity generated by the front wing is taken account of. Interferences between the fuselage and the forward wing are considered as well. The average downwash gradient $\mathrm{d}\varepsilon/\mathrm{d}\alpha$ is estimated according to method two in section 4.1.1 of [14]. The related equations for calculating the lift curve slope of the whole aircraft as well as the downwash gradient are extensive and therefore not shown in this paper. The interested reader can find the relations in [9].

The evaluation of Eq. (22) is done with the help of a spreadsheet which is further referred to in section 3.5.

3.5 Evaluation

The conditions derived in section 3.3 are evaluated with the help of a spreadsheet. Input parameters are the geometry of the wing configuration and the aerodynamic parameters for cruise flight. Like this it is possible to assess different geometric and aerodynamic configurations. Flight phases other than cruise have yet to be investigated.

It is desired to have the greatest possible CG envelope, which means that the stability limit should be situated as far aft as possible while the control limit should be situated as far forward as possible.

3.5.1 Stability Limit

The stability condition [Ineq. (19)] depends on the gradient $\mathrm{d}C_{L,2}/\mathrm{d}C_L$ and on the modified volume coefficient $\bar{V}'$. A high value for $\mathrm{d}C_{L,2}/\mathrm{d}C_L$ is beneficial for static longitudinal stability. Under practical conditions $\mathrm{d}C_{L,2}/\mathrm{d}C_L = 1$ can already be considered to be a very satisfying value. $\mathrm{d}C_{L,2}/\mathrm{d}C_L$ mostly depends on the sweep of both wings. The sweep of the forward wing needs to be high. For example, reducing the sweep angle from 35° to 0° results in a reduction of about 10 % for $\mathrm{d}C_{L,2}/\mathrm{d}C_L$. The aft wing is swept forward. Here a lower amount of sweep is favorable. For example, a reduction of sweep from -25° to 0° yields an increase of about 6 % for $\mathrm{d}C_{L,2}/\mathrm{d}C_L$. These values apply for the medium range box wing aircraft presented in section 4.

The stability limit highly depends on the modified volume coefficient $\overline{V}'$. A high value improves stability. $\overline{V}'$ increases with increasing l' and hence when the wings are placed far apart from each other.

Looking at the trim condition [Ineq. (20)] it becomes obvious that the important parameters are the lift coefficient of the aft wing at zero total lift $(C_{L,2})_{C,L=0}$ and the modified volume coefficient $\overline{V}'$ which has a positive value. For attaining static longitudinal stability the aft wing has a lower lift coefficient than the forward wing, hence $(C_{L,2})_{C,L=0} < 0$ (see Fig. 7). This results in the left hand side of Ineq. (20) being positive, even if pitching moment coefficients are negative.

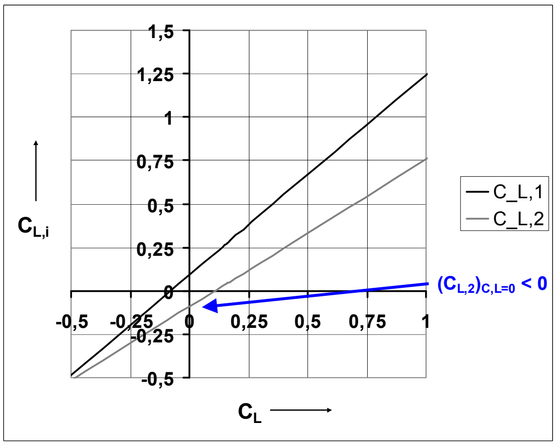

Fig. 7 Lift coefficients of the individual wings vs. total lift coefficient of the box wing aircraft

Manipulating the pitching moment coefficient of the wings (or possibly the fuselage) is not considered at this stage, but will be discussed in section 3.6. According to Eq. (21) $(C_{L,2})_{C,L=0}$ depends on the lift coefficient of the aft wing at the investigated flight state as well as the total lift coefficient at this flight state and the gradient $dC_{L,2}/dC_L$. A low value for the lift coefficient of the aft wing $C_{L,2}$ helps to comply with the trim condition. For a given total lift coefficient this means that the ratio $C_{L,1}/C_{L,2}$ needs to be high.

For conventional tail aft configurations the positive zero lift pitching moment is provided by the horizontal stabilizer which produces a downward force and thus lets the aircraft pitch up.

3.5.2 Control Limit

For having a stable and controllable aircraft the control limit has to be situated in front of the stability limit. The more forward the control limit, the larger the CG envelope. Thus the sum of the right hand side of Ineq. (18) has to have the smallest possible value. In the beginning it is assumed that both wings have equal mean aerodynamic chord lengths and wing areas. Thus $\overline{V}'$ can only be changed with the help of the longitudinal distance of both wings l'. This parameter is however influenced by the sweep of both wings, their integration with regard to the fuselage and the tail as well as a feasible winglet sweep. Consequently changing l' requires a complete redesign of the aircraft. So the only parameter to be freely manipulated in Ineq. (18) is the lift coefficient of the aft wing. All of the other parameters are defined by the flight state. Consequently the ratio $C_{L,2}/C_L$ needs to be low (or $C_{L,1}/C_{L,2}$ needs to be high), as it was already desired at the end of section 3.5.1.

3.5.3 CG Envelope Diagram

It is possible to plot the CG envelope depending on the modified volume coefficient of the aft wing. The relating diagrams are shown in Figs. 8, 9 and 10 for different values of $C_{L,2}/C_{L,1}$. They apply for the medium range box wing aircraft presented in section 4 under cruise conditions with maximum take off weight and maximum payload. The CG envelope is the region forward of the stability limit and aft of the control limit. In Fig. 8 the ratio $C_{L,1}/C_{L,2}$ is unity. It can be seen that the box wing aircraft is unstable because the control limit is aft of the stability limit. Having a value lower than one (Fig. 9) moves the control limit further aft which deteriorates stability. The diagram for a properly chosen ratio of $C_{L,1}/C_{L,2}$ is depicted in Fig. 10. The relating CG envelope is indicated with the green line and is about 48 % MAC, which equals 24 % MAC for a conventional reference aircraft. The CG of the aircraft is within the allowable limits taking account of a static margin of about 5 % MAC in terms of a conventional wing. The static margin is indicated with the yellow line. In Fig. 10 the CG envelope would increase with higher values for

$\overline{V}'$. Hence the longitudinal distance between both wings should be increased for a larger CG envelope.

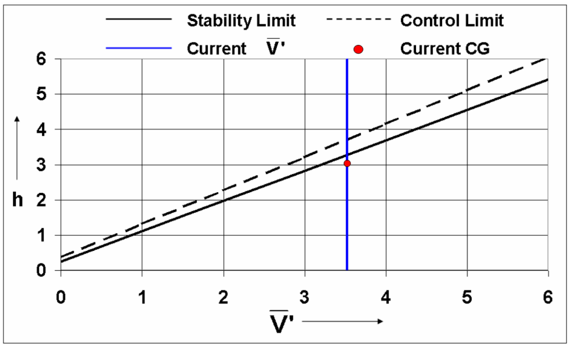

Fig. 8 CG envelope diagram for an unstable box wing aircraft ($C_{L,1}/C_{L,2} = 1$)

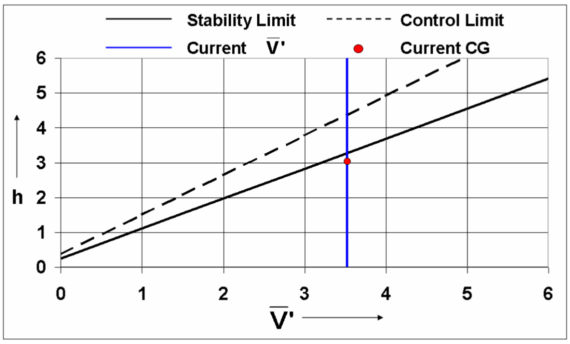

Fig. 9 CG envelope diagram for an unstable box wing aircraft ($C_{L,1}/C_{L,2} < 1$)

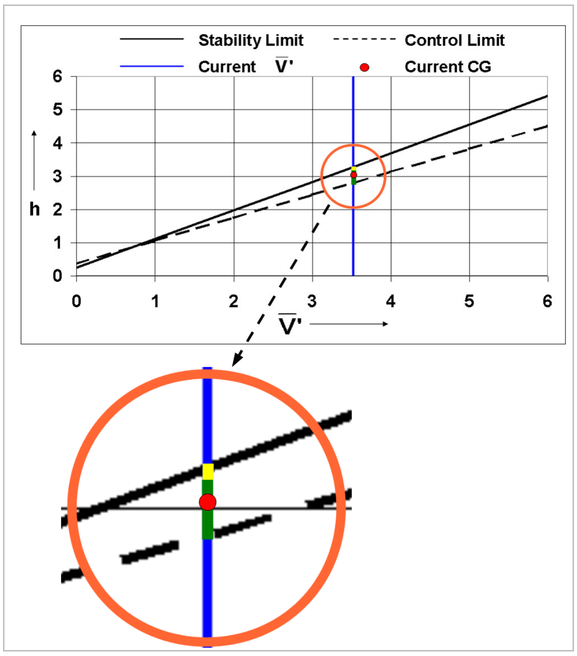

Fig. 10 CG envelope diagram for a stable box wing aircraft ($C_{L,1}/C_{L,2} > 1$)

For conventional tail aft aircraft the CG envelope diagram is used in order to determine the required $\overline{V}'$ for a desired CG envelope. Like this it is possible to size the horizontal stabilizer independently from the other aircraft components. This approach is not possible for a box wing aircraft. Here the value for *l'* and the resulting $\overline{V}'$ is the result of the designed configuration. Based on the design the CG envelope diagram is drawn and it is checked whether the aircraft's CG is within the allowable limits. If this is not the case, the aircraft has to be completely redesigned because the design of the wings, the fuselage, the tail and the resulting CG position all depend on each other. For conventional tail aft configurations it is possible to move the wing independently if it does not suit the CG position.

3.6 Alternative Ways of Attaining Stability

In section 3.5 the option of having unequal lift coefficients is favored for attaining static longitudinal stability. However, there also exist alternative options which have not been mentioned yet or which were intentionally excluded in order to explain the main idea in simple terms first.

It is possible to combine wing twist and sweep so that the wing has a positive pitching moment. A part of the wing configuration in front of the CG has to produce a high amount of lift (e.g. the center region of the forward wing) while a part aft of the CG produces less lift or even a downward force (e.g. center region of the aft wing). This principle is also applied to flying wings where the wing tips mostly generate a downward force. This approach is not considered for the design of the medium range box wing aircraft because it causes the span wise lift distribution to deviate from the optimum.

Another option is an adaption of the airfoils. With the help of reflexed airfoils whose rear sections are cambered upwards it is possible to have a positive pitching moment without applying twist and sweep as described above. However, it is unlikely that such airfoils are applicable to a transonic transport aircraft. In addition these airfoils will not provide a high glide ratio. Designing the fuselage with a reflexed rear section is also possible, as it is shown in [15], but the question of applicability arises again.

Another possibility is a control surface at the tail of the aircraft. In [16] a surface at the rear end of the fuselage is proposed. This option seems to be promising. A new formulation of the equilibrium of moments would be necessary to evaluate the effects regarding stability and controllability. The inner part of the aft wing could be equipped with upward deflecting elevators. However, this is nothing else as airfoils whose rear sections are cambered upwards. Substantial additional drag would be the consequence. An additional horizontal stabilizer independent from the aft wing could be a possibility or using a V-tail with symmetrically upwards deflected elevators (see Fig. 11). Further investigations would need to show if it is better to accept resulting trim drag than to accept a higher than minimum induced drag due to a lift distribution where $L_1/L_2 > 1$ according to Fig. 4.

4 Application to a Medium Range Box Wing Aircraft

4.1 Design Approach

The design of the medium range box wing aircraft is based on a reference aircraft comparable to the Airbus A320. Both aircraft have the same design mission. Like this it is possible to compare the characteristics of both aircraft in order to assess the potential of the box wing configuration. The design study is presented in [9] and [17]. For having a valuable comparison both aircraft are supposed to have the same total wing area and the same wing span. Both wings of the box wing aircraft have the same reference area. Because most of the investigations are preliminary the found design still leaves room for optimization.

4.2 Implementation of Stability Requirements

As discussed static longitudinal stability and controllability is attained with the help of unequal lift coefficients of both wings. The ratio $C_{L,1}/C_{L,2}$ is about 1,74 which was determined for cruise conditions. If an average lift coefficient of about 0,75 is assumed for cruise, the lift coefficient of the forward wing would be 0,96 and that of the aft wing 0,55. The resulting CG envelope is about 48 % MAC, which is equivalent to an envelope of 24 % MAC for a conventional reference wing. This magnitude is only applicable for aircraft being well balanced with regard to their CG position. Aircraft with a different weight distribution require a larger CG envelope. For the box wing aircraft this would mean a further increase of the ratio $C_{L,1}/C_{L,2}$, which is not feasible taking account of the cruise lift coefficients of the individual wings. Consequently the box wing aircraft needs to be well balanced as well. This requires the individual CGs of the airframe, engines, fuel and payload all to be located approximately at the same position. Hence the CG shift is minimized for different payload and fuel quantities. The most forward position of the wing is limited by the required clearance to the front door. A forward swept vertical tail brings the wing in a symmetric position with respect to the fuselage. A compact fuselage also minimizes the CG shift for different loading conditions. Since the CG is close to the center of the cabin the CG shift proceeds almost symmetrically during loading. The resulting cabin layout includes seat rows with eight seats abreast and two main aisles. The increased cross sectional area allows for an accommodation of standard LD3 containers. The fuselage is formed as a drop which is an efficient aerodynamic shape. Detailed investigations of the optimum fuselage fineness ratio (fuselage length to diameter ratio) can be found in [18]. Fig. 12 of [18] shows a minimum drag of the fuselage relative to cabin surface for a fineness ratio of about 7.5. With a fineness ratio of 5,8 the proposed box wing aircraft is not too far from this optimum. The fuselage is lighter than the reference fuselage because of its higher diameter and decreased length. The resulting aircraft layout is shown in Fig. 11.

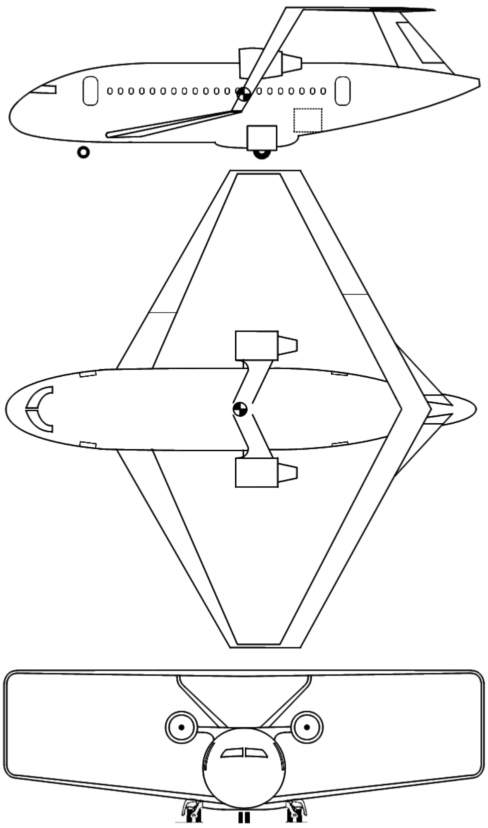

Fig. 11 Three view drawing of the medium range box wing aircraft

Another design feature of the aircraft is the V-tail which is also proposed in [5]. With its help the aft wing is supported by two surfaces what increases its stability. Mainly the V-tail is supposed to function as vertical stabilizer, but its angular surfaces also provide the possibilities of a horizontal stabilizer. It could help to achieve the positive zero lift pitching moment which is needed to comply with the trim condition [Ineq. (20)]. However, this would require a new analysis.

4.3 Decrease of Aerodynamic Efficiency

With the help of the ratio $C_{L,1}/C_{L,2}$ it is possible to determine the increase of induced drag according to biplane theory [Eq. (7)]. Since both wings of the box wing aircraft have the same reference area and it is assumed that they are exposed to the same dynamic pressure, the ratio of lift coefficients is the same as the ratio of total lift L_1/L_2. Hence it is possible to apply Eq. (7). With the given ratio of 1,74 (section 4.2) $D_i/D_{i,min}$ = 1,034 is the result, so there is a 3,4 % increase of induced drag because of the unequal lift of both wings. According to Eq. (24) below this is equal to a 3,4 % decrease of the span efficiency factor, provided that all of the other parameters in Eq. (24) are constants.

$$D_{\mathrm{i}} = \frac{L^2}{q \cdot \pi \cdot b^2 \cdot e} . \qquad (24)$$

Applying Eq. (25) for the determination of the maximum glide ratio E_{max} this results in a 1,7 % decrease of the maximum glide ratio.

$$E_{max} = \frac{1}{2}\sqrt{\frac{\pi \cdot A \cdot e}{C_{D,0}}} . \qquad (25)$$

$C_{D,0}$ is the zero lift drag coefficient of the aircraft. For the designed medium range box wing aircraft a drop from 20,7 to 20,4 results for E_{max} because of the unequal lift of both wings. Because of that the aircraft consumes about 110 kg more fuel for the reference mission than with minimum induced drag.

It needs to be emphasized that these numbers result when applying biplane theory, as it is also done in [2] and [5]. But from [10] and [11] it can be understood that biplane theory may not be applicable to box wings in terms of induced drag calculation because the theory does not take account of the special characteristics of closed wing systems. According to [10] and [11] there should be no increase of induced drag for ratios of L_1/L_2 being higher than one. So it seems that biplane theory gives an upper limit of the increase of induced drag. However, the efficiency loss according to biplane theory is small. Hence it is unlikely to have a significantly decreased induced drag with the options presented in section 3.6.

The determination of the induced drag of a box wing configuration with the help of simple equations may not produce results accurate enough. For a more detailed assessment of the induced drag of the final configuration it is necessary to use computational fluid dynamics, preferably vortex lattice methods.

5 Conclusion

In this paper the characteristics concerning static longitudinal stability and controllability of box wing aircraft and the resulting effects regarding aerodynamic efficiency were investigated. It was shown that it is possible to determine the induced drag of an optimally loaded box wing aircraft with the help of literature data. According to biplane theory one condition for optimum loading is that both wings generate the same amount of lift. From the assessment of static longitudinal stability it was reasoned that this condition is most likely violated.

For assessing static longitudinal stability and controllability the theoretical foundations were presented. These were applied to the box wing configuration which resulted in a formulation of the allowable CG envelope, based on the negligence of thrust and drag forces as well as the assumption of horizontal and non-accelerated flight without an additional horizontal stabilizer or control surface deflection. It was discovered that having equal lift coefficients at both wings leads to an unstable aircraft provided that the zero lift pitching moment of the wings is negative. For a stable aircraft the ratio $C_{L,1}/C_{L,2}$ has to reach a certain value higher than one. The higher the value, the larger is the CG envelope of the aircraft. Depending on the flight state the lift coefficient of the forward wing could become problematic. It is also possible to attain stability without increasing the ratio $C_{L,1}/C_{L,2}$. In this case the combined zero lift pitching moment of the wings and the fuselage needs to be positive. A compromise between a manipulation of the wing/fuselage zero lift pitching moment and a lower ratio $C_{L,1}/C_{L,2}$ is also possible. Adapting the wing geometry for attaining stability might be in conflict with other design requirements.

The application of the found results was demonstrated with the help of a medium range box wing aircraft based on the Airbus A320. The wings were supposed to have a value of -0,1 for the zero lift pitching moment. The ratio $C_{L,1}/C_{L,2}$ was determined to be 1,74 for cruise which provides a CG envelope of 48 % MAC. This is equal to an envelope of 24 % MAC for a comparable and conventional reference aircraft. A larger envelope would require a higher value for $C_{L,1}/C_{L,2}$ which does not seem to be feasible. Therefore the layout of the aircraft is well balanced regarding the CG position. The empennage was chosen to be a V-tail which could act as additional horizontal stabilizer. In this way the CG envelope could be increased for a given $C_{L,1}/C_{L,2}$ ratio or the $C_{L,1}/C_{L,2}$ ratio could be decreased for a given CG envelope. A profound assessment of this possibility has yet to be made.

The decrease of the span efficiency of the medium range box wing aircraft because of stability requirements was determined according to biplane theory. Its span efficiency factor is decreased by 3,4 % which results in a 1,7 % decrease of the maximum glide ratio. This means an increase in fuel burn of about 1 % for the reference mission, which is about 110 kg. However, judging from the aerodynamics of closed wing systems there may not even be an increase in induced drag and hence in fuel consumption when the box wing aircraft is stabilized with a ratio $C_{L,1}/C_{L,2} > 1$.

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