DEFINITION AND DISCUSSION OF THE INTRINSIC EFFICIENCY OF WINGLETS

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ABSTRACT

Three simple equations are derived to define the "intrinsic aerodynamic efficiency of winglets" independent of the horizontal extension of the winglet and independent of the winglet's (relative) height. This "intrinsic aerodynamic efficiency" allows a quick comparison of purely the aerodynamic shape of winglets independent of the selected size chosen for a certain aircraft installation. The intrinsic aerodynamic efficiency is calculated in 3 steps: STEP 1: The relative total drag reduction due to the winglet is converted into an assumed contribution of the winglet only on the span efficiency factor. STEP 2: If the winglet also increases span, its performance is converted into one without the effect of span increase. STEP 3: The winglet's reduction in induced drag is compared to a horizontal wing extension. If the winglet needs e.g. to be three times longer than the horizontal extension to achieve the same induced drag reduction, its "intrinsic aerodynamic efficiency" is the inverse or 1/3. Winglet metrics as defined are calculated from literature inputs. In order to evaluate winglets further, the mass increase due to winglets is estimated in addition to the reduction of drag on aircraft level and fuel burn.

KEYWORDS: *wingtip, winglet, induced drag, wing mass, aircraft design*

NOMENCLATURE

Upper Case Latin

A – aspect ratio, $A = b^2/S$
A – coefficient in $D = AV^2 + BV^{-2}$
B – coefficient in $D = AV^2 + BV^{-2}$
C – aerodynamic coefficient
D – drag
L – lift
S – surface area
V – true airspeed

Lower Case Latin

b – wing span
c – chord
d – diameter
e – span efficiency factor; Oswald factor
g – earth acceleration
h – height of winglet; span increase
k – factor
m – mass

Greek Symbols

Δ – difference, Delta
ρ – air density

Subscripts

beef – "beefing up" of wing structure
co – *cross over*
CR – *cruise*
D – drag
D0 – *zero-lift drag*
Di – induced drag
eff – effective
F – fuel
h – horizontal
i – induced
L – lift
md – minimum drag
MTO – maximum take-off
MZF – maximum zero fuel
ref – reference (A/C without winglet)
t – tip of wing
W – wing
WL – winglet

1 INTRODUCTION

1.1 Motivation, Aim and Scope

Most passenger aircraft today have winglets. Winglets look stylish (Fig. 1). Winglets are used to advertise the airline's logo like on the vertical tail (Fig. 2). Often winglets have fancy names. The B737NG was equipped in 2014 with "Split Scimitar Winglets". The B737 MAX has "AT Winglets", where "AT" stands for "Advanced Technology". "AT Winglets" are also split winglets [5]. Airbus calls the new blended winglets on the A320neo "Sharklets" (2012). Sharklets are nothing more than "blended winglets" introduced on the B737NG already in 2001. Blended winglets simply combine the horizontal wing with the vertical winglet via a certain radius. A patent about "blended winglets" was already published in 1994 [6].

Figure 1: Different winglets on passenger aircraft:
A321 Sharklet [1], A350XWB Blended Winglet [2] and B737 MAX AT Winglet [3]

Figure 2: Airline Logos on Winglets:
Southwest Airlines [4], Tui Fly, Air Berlin, Ryanair (own pictures)

Manufacturers make various claims about the performance gain achieved due to their winglets. The information is conveyed with press releases in an advertisement style. More official data from manufacturers is usually missing and numbers in press releases may not necessarily match up and make sense (see Appendix). When we look at claims about winglets in scientific literature we find that claims even here do not match up (see Chapter 1.3, 1.4 and 3.3). These are apparently already difficult questions: 1.) What is better, a near vertical winglet or a span increase? It is said that the magnitude in performance gain "depend[s] strongly on the design details of the baseline airplane and the [tip] device" [7]. Therefore: 2.) If two winglets differ in size or other geometric parameters, how can the winglet's aerodynamic quality be measured and how can the two winglets still be compared? A certain reduction in induced drag coefficient (at constant lift coefficient) can always be reached, if the winglet is high enough and installed with a cant angle such that wing span is also increased. However: 3.) Is there an overall benefit in drag and fuel burn due to the winglet? This paper tries to answer these questions and more. Its aim is to raise awareness of what winglets can achieve and what not. The paper will help readers to discuss winglet performance based on sound scientific and practical knowledge. The equations given here can also be used to make a preliminary aircraft design including winglets. Detailed aircraft design with winglets will have to make use of Computational Fluid Dynamics (CFD) for aerodynamics and loads and Finite Element Methods (FEM) for wing mass estimation. CFD and FEM are beyond the scope of this paper.

1.2 Basic Aerodynamics of Winglets

Winglets are included in a design or added to an existing design to reduce drag. Different suggestions are made in the literature to **classify drag**. These suggestions need to take into account what methods are finally available to calculate the different drag components. There is generally agreement to distinguish on the first level of the hierarchy between *zero-lift drag* (drag independent of lift) and *induced drag* (drag due to lift; strongly depending also on Mach number). Having said this, we need to state that zero-lift drag also depends on lift and induced drag also depends on zero-lift drag, but this is beyond what should be discussed here. Zero-lift drag can be broken down into *profile drag*, *wave drag* (strongly depending on Mach number), *interference drag*, and *miscellaneous drag* (trim drag and additional drag). Profile drag can in turn be subdivided into *skin-friction drag* and *pressure drag*. Details are explained and calculation methods are given e.g. here: [8][9][10].

Winglets reduce induced **drag** (see below), but add zero-lift drag (due to additional surface area) and interference drag (due to interference of the flow around the winglet and the flow around the wing at the point where the two surfaces meet – unless the wing blends well into the winglet). At transonic speeds the winglet will experience wave drag. The winglet should be swept to limit wave drag.

Several explanations have been put forward to explain **why winglets reduce induced drag**. A valid explanation is this: "To create ... lift, the wing pushes downward on the air it encounters and leaves behind a wake ... forming two large vortices" (Fig. 3). The energy required to create this wake is reflected in the airplane's induced drag (also called *vortex drag*). The basic method by which the vortex drag may be reduced is to increase the horizontal or vertical extent of the wing. By increasing the wing dimensions, a larger mass of air can be affected by a smaller amount to produce a given lift, and this leads to less energy in the wake and lower induced drag [11]. A **classical derivation** [12][13] models the mass of air affected by the wing as a cylinder with diameter, $d = \sqrt{2}\, b$ and length, $l = V\, t$, with flight speed, V and time, t (t cancels out later in the derivation) (Fig. 3). This yields – in the ideal case of an elliptical span loading – an equation for the induced drag

$$D_i = \frac{L^2}{\frac{1}{2}\rho V^2\, \pi\, b^2} \tag{1}$$

for a wing with geometrical span, b. In the real case for a non-elliptical span loading, the span efficiency, e has to be considered and

$$D_i = \frac{L^2}{\frac{1}{2}\rho V^2\, \pi\, e\, b^2} \quad . \tag{2}$$

It can be thought of the winglets pushing the wing tip vortices out further away from the tip, so that a larger effective span, b_{eff} results affecting the air mass and

$$D_i = \frac{L^2}{\frac{1}{2}\rho V^2\, \pi\, e\, b_{eff}^{\,2}} \quad . \tag{3}$$

Decisive according to Faye [14] is "the length of the [trailing edge] TE that sheds the vortices". "Winglets increase the spread of the vortices along the TE". According to the model depicted in (Fig. 3, right) it seems clear that a long horizontal trailing edge influences a larger air volume than a trailing edge of the same length that bends up at the wing tip. So, just from this geometrical consideration, it seems clear that a horizontal wing extension is more effective than a winglet.

A more detailed calculation of winglets is possible with the **Trefftz-plane theory** [15]. It tells us that we can reduce the ideal induced drag by increasing the vertical height of the lifting system, as well as by increasing the horizontal span. If we consider a wing system that must fit within a given rectangular box, Trefftz-plane results show that the lowest-drag configuration is the box wing, which has lifting surfaces along all four sides of the box. Favorable configurations are those that reach into corners of the box and seal at least some of the four sides of the box. A winglet with $h/b = 0.2$ reduces induced drag to 82% [7] or $b_{eff}/b = 1.1$, however, an equivalent span increase would result in $b_{eff}/b = 1.4$.

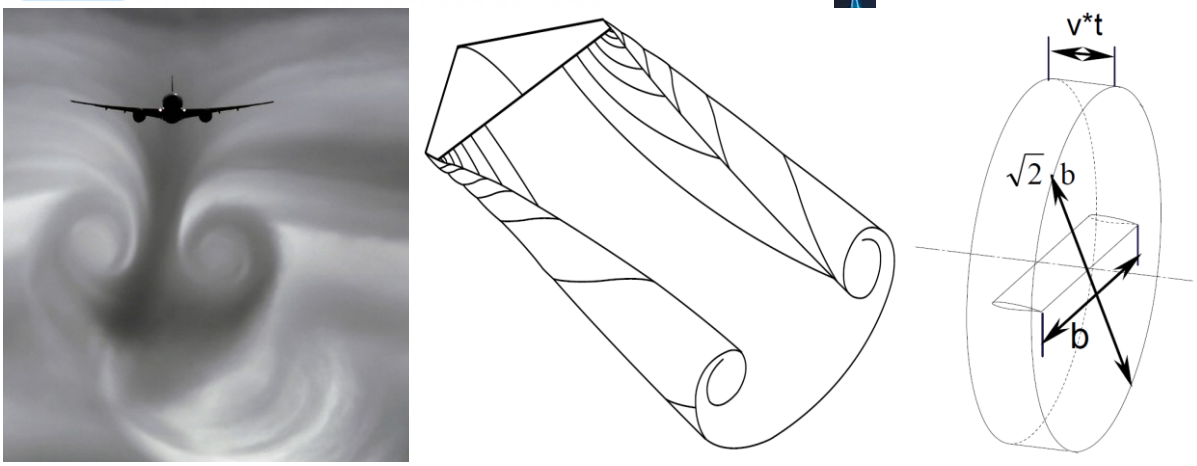

Figure 3: The vortex wake behind a lifting wing (left [11] and middle [7]). Classical derivation of induced drag: A cylinder of air assumed to be affected by the wing (right).

McLean [7] is debunking myths and misconceptions regarding the vortex wake and wingtip devices:

- "The vortex cores are often referred to as 'wingtip vortices', though this is a bit of a misnomer." "The vorticity that feeds into the cores generally comes from the entire span of the trailing edge, not just from the wingtips." This "leads us to think we can influence the induced drag by acting only on a very small part of the flow." "There is no credible evidence that any such device can provide a reduction in induced drag, beyond what can be explained as the result of an increase in physical span when the device is added."
- "There is a common misunderstanding that a wingtip device reduces drag by producing thrust on the surfaces of the device itself. This line of thinking is wrongly based on the flowfield that would be there in the absence of the winglet. The real flowfield is [however] altered considerably if the winglet is properly loaded."

1.3 The "Classic" on Winglets in the Literature

Probably most cited when it comes to winglets is NASA-TN-D-8260 by Richard T. Whitcomb. This report was one result of the Aircraft Energy Efficiency (ACEE) program initiated by NASA and the Department of Energy (DOE) after the fuel crisis. Whitcomb was an outstanding engineer. He is listed in the "NACA and NASA Langley Hall of Honor" [16] Among many other inventions he is honored for the winglet: "Wing-tip vertical end-plates had been used in efforts to reduce drag many years before Whitcomb's design efforts, but his ingenious and detailed analysis led to special tailoring of such devices, which proved to significantly reduce drag at cruising speeds. He called his invention 'winglets' " [17]. Whitcomb, 1976:

> *"For the [two] configurations investigated the winglets reduce the induced drag by about 20% with a resulting increase in wing lift-drag ratio of roughly 9 percent ... This improvement in lift-drag ratio is more than twice as great as that achieved with the comparable wing-tip extension." "A comparison ... with ... a wing-tip extension ... results in approximately the same increase in bending moment at the wing-fuselage juncture as did the addition of the winglets."* [18]

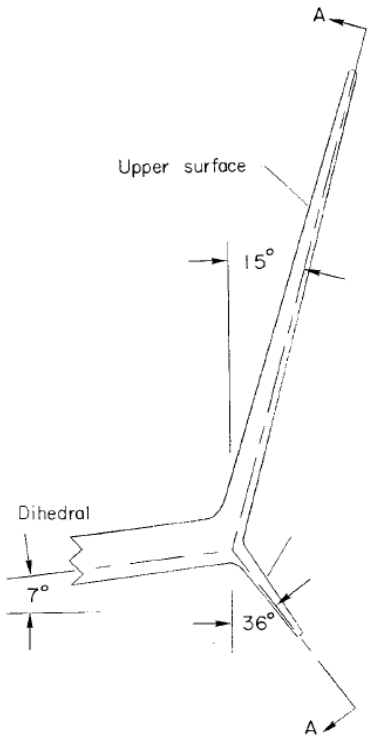

Figure 4: Whitcomb's winglet [17]

Many have taken Whitcomb's statement without looking at the geometry that got analyzed. Whitcomb compares (incorrectly) two arbitrary chosen wing tip devices that are not comparable:

a) a horizontal tip extension of h_h = 0.076 m and b) a near vertical split winglet c_t = 0.203 m up and 0.23·c_t down and hence with total height h = 1,23·c_t = 0.250 m, (Fig. 4). Similar criticism can be found in the literature:

- "It should be noted that these results were obtained for a particular wing, winglet, and wingtip extension. Potential drag savings and moment distributions depend strongly on the geometry of the surfaces." "The tip modification [of the B747-400] increased the cruise L/D approximately 4 percent (less than half of the upper limit of 9 percent suggested by wind tunnel tests in the ACEE program ...), with much of the improvement coming from the span extension." [11]
- "Whitcomb's ... results of wind-tunnel tests [were] comparing configurations that were not comparable." "[His] rule of thumb has not been borne out by studies since then." [7].

1.4 More from Fundamental Literature

It has always been the question how a winglet compares to a span extension with respect to drag reduction and mass increase. As a rule of thumb the **ratio** between winglet height and comparable span extension is discussed.

- Larson, 2001 [19] (original source unknown):
  "One rule of thumb says that for an increase in wing-bending force equal to that of a one-foot increase in span, a wing's structure can support a three-foot winglet that provides the same gain as a two-foot span extension." That is: Same drag reduction at half the mass and winglet of **ratio 1.5**.
- Jones, 1980 [20] quoting Prandtl, 1933 [21]:
  "[With] a constraint on the integrated bending moments, ... a 10-percent reduction of induced drag can be achieved by a 10-percent increase of wing span accompanied by a more highly tapered loading."
- Jones, 1980 [20]:
  "The same result can be obtained by a 15-percent vertical extension. Thus, it appears that with ideal wing shapes similar reductions of induced drag can be achieved by either horizontal or vertical tip extensions." That is: Same drag reduction at same mass and winglet of **ratio 1.5**.
- McLean, [7] quoting Jones, 1980 [20]:
  "The calculations indicate that horizontal span extensions and vertical winglets offer essentially the same maximum induced-drag reduction when the spanloads are constrained so that there is no increase in 'structural weight'. They also indicate that to achieve a given level of drag reduction, a vertical winglet must be nearly twice as large as a horizontal span extension." That is: Same drag reduction at same mass and winglet of **ratio 2.0**. As we will see below this last rule of thumb is the one that comes quite close to the truth.

1.5 More Literature

NASA-TN-D-8260 by Richard T. Whitcomb was followed by NASA-TP-1020 in 1977 [22] it includes many diagrams about span efficiency factor increase versus wing root bending moment increase. Among the many aircraft design textbooks it seems that only Gudmundsson (2014) [23] offers a design method for winglets. The method also is used in a case study retrofitting winglets to the Dassault Falcon 10 business jet [24]. Many papers report about a detailed aerodynamic analysis of a winglet of a specific geometry, however, with little information about how these findings can be generalized. One such study should be mentioned [25]. It compares a certain winglet with a raked tip against the wing reference in the wind tunnel and with CFD. Quite a complete overview about passenger aircraft and their winglets is given in [11], but as the matters stand, also with little data.

2 WINGLET WISDOM

2.1 Fundamentals

- Winglets reduce induced drag, but add zero-lift drag due to the fact that they add wetted area to a given wing area. In contrast, a span extension (aspect ratio increase) will be done at constant wing area, because also the new area at the tip contributes to the lift and as such area added on the wing at one location can be subtracted at another location.

- Drag, D is a function of true air speed, V. Drag is calculated from $D = AV^2 + BV^{-2}$. This is the speed polar. The *Minimum Drag Speed*, $V_{md} = (B/A)^{1/4}$. It depends on the zero-lift drag coefficient C_{D0}, span efficiency factor e and aircraft mass in cruise m_{CR}. A increases with C_{D0}. B increases with m_{CR} and decreases with e. For a good winglet that does not increase C_{D0}, does not increase wing mass and hence does not increase m_{CR}, drag D is reduced and V_{md} is reduced as well. [26]
- The *Crossover Speed*, V_{co} on the speed polar is the speed at which the speed polars of the aircraft without winglets and with winglets intersect. With the Crossover Speed it is easy to make the tradeoff between the zero-lift drag penalty and the induced-drag benefit. Below this speed, winglets are beneficial, whereas above it they are detrimental. Flying at the Crossover Speed means to fly at a speed where the benefit in induced drag due to winglets is equal to the zero-lift drag penalty. The more the induced drag can be reduced for a given increase in zero-lift drag, the higher the Crossover Speed and the more useful the winglet, because of its wide useful speed range. (See also [27])
- Winglets work best at high lift coefficients (i.e. speeds speed below the Crossover Speed). This is the case for take-off, climb, approach and landing.
- Winglets are detrimental at very low lift coefficients which occur at high cruise speed combined with low altitude and low aircraft mass (small payload and small fuel quantity).
- A winglet adds wing bending to the wing loads. A span increase adds wing bending to the wing loads in much the same way as the winglet *and* adds shear forces. Beefing up shear webs does not generally add much wing mass, but it could be expensive in retrofit applications [7].
- For unlimited wing span, a span increase is in almost all situations superior to a winglet. For passenger aircraft raked tips have been used [7]. For *extremely* high aspect ratios (sailplanes) winglets where found more beneficial than a further span increase [27].
- For limited wing span, however, the winglet is a good way to reduce induced drag. Wing span limits at airports are due to the "FAA Airplane Design Group" and the "ICAO Aerodrome Reference Code". Both are identical with respect to the maximum wing span definition (Table 1).

Table 1: ICAO Aerodrome Reference Code [28]

code letter	wingspan
A	< 15 m
B	15 m but < 24 m
C	24 m but < 36 m
D	36 m but < 52 m
E	52 m but < 65 m
F	65 m but < 80 m

2.2 Pros and Cons

- Benefits of winglets:
  - Induced drag reduction
  - Larger lift curve slope (due to larger effective span)
  - Reduced fuel burn
  - Increased payload
  - Increased maximum range
  - Reduced takeoff field length due to improved second segment climb
  - Meet gate clearance with minimal performance penalty
  - Appearance and product differentiation
  - Increased residual aircraft value (add 700000 USD for a B737NG at installation and depreciate together with aircraft over time) [29]
- Negative factors of winglets:
  - Increased wetted area leading to increased zero-lift drag
  - Junction flows leading to increased interference drag
  - Mass increase due to the device itself
  - Mass increase due to the mass of attachment fittings
  - Increased difficulty in cross-wind landings
  - Increased loads on the wing with side slip
  - Increased tendency to flutter due to added mass at the wing tips

- Mass increase to the existing wing structure ("beefing up") due to larger static loads, more demanding flutter loads and fatigue requirements
- Increased costs for the manufacturer (non-recurring costs and recurring costs)
- Increased costs for the airline (purchase costs, maintenance costs – 6.5 hours scheduled maintenance per year [29])
- Increased development risk
- Split winglets (that extend below the wing) are prone to damage from ground service equipment (this could lead to unscheduled maintenance costs, delays and cancellations)

Compare also with [30].

2.3 General Hints

These hints are largely based on [7].

- The induced-drag reduction that can actually be achieved in most applications typically falls significantly short of the ideal.
- When a winglet is included in the design of an all-new wing, the structural mass penalty of "beefing up" the wing structure must be paid in full. On an existing airplane, flight testing will sometimes have established that the wing has excess structural margin that can be "used up" by the addition of a tip device.
- If the twist distribution of the existing wing was optimized for operation without a winglet, the benefit available from the addition of a winglet will usually be substantially less than it would have been if the wing could have been re-optimized.
- When horizontal span extensions and vertical winglets are designed, it is found that they offer the same induced-drag reduction when the same wing bending load occurs. So, in terms of the trade between drag reduction and mass increase (to the wing structure), horizontal span extensions and vertical winglets have almost the same performance potential.
- To achieve the same induced-drag reduction, a vertical winglet must be considerably higher than the span increase. This is expressed with the parameter k_{WL} (see Chapter 3.1). Note: This adds more winglet mass than horizontal tip mass.
- The percentage drag reduction shows a diminishing rate of return with increasing device size.
- The percentage mass increase tends to be roughly linear with size.
- Because of the mass increase, the percentage fuel-burn reduction is less than the percentage drag reduction.
- The increase in maximum range depends on what is limiting the range. If the range is limited by maximum take-off mass (MTOM), the mass increase due to the winglet will subtract directly from the fuel that can be carried, and the increase in range may be very small. Only if the aircraft takes-off from a short runway or is climb-limited winglets help to carry more mass (and fuel) out of the airport which extends range.
- Tip devices of a wide variety of types seem to have very similar potential with regard to the drag/mass trade (at the same value of h/b).
- Winglets on the upper wing increase effective dihedral; this needs to be accounted for in a new wing design; retrofits have to cope with (known) consequences.

2.4 Hints to Detailed Design

These hints are largely based on [7].

- *Part-chord winglets* follow the strategy of integrating an outboard chord distribution consistent with the desired span load with an existing trapezoidal wing that has more chord than it needs at the tip (e.g. due to aileron integration).
- *Blended winglets* have no discontinuous change in chord at the junction as there would be with a conventional part-chord winglet, but within the blending region, the chord decreases rapidly and smoothly, so that the chord distribution from there out is similar to that of a part-chord winglet.
- Any retreat from the corners of the box (due to "blending" with a radius in the junction between the winglet and the wing) increases ideal induced drag, but avoids interference drag associated with sharp corners and reduces wetted area a little.
- For other blended winglet parameters (e.g. the winglet radius), see equations in [6].
- *Split winglets* have the winglet height split into two equally separated winglets above and below the wing, each winglet span is only half the size and bending moments are half the original. If the

winglet's chord is sized to the load carried, the split winglet needs only half as much chord as the single winglet and will have half the mass. However, split winglets produce only about 90% of the induced drag reduction compared with a standard winglet of the same total height.

3 ESTIMATING WINGLET AERODYNAMICS

3.1 Derivation of the Intrinsic Efficiency of Winglets: $1/k_{WL}$

The simplest approach in understanding winglets is to consider the effect of the winglets equal to that of a wing that prolongs its span with the size of the winglets, as in Fig. 3. See also [31], [25].

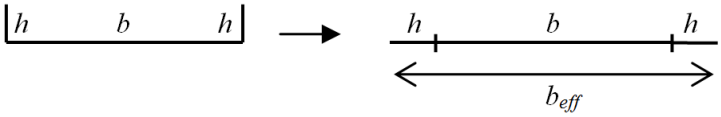

Figure 5: Simple geometrical consideration for winglets evaluation

The following relations can be written

$$\frac{b_{eff}}{b} = 1 + 2\frac{h}{b}$$

$$C_{D,i} = \frac{C_L^{\,2}}{\pi A e}$$

$$C_{D,i,WL} = \frac{C_L^{\,2}}{\pi A_{eff} e} = \frac{C_L^{\,2}}{\pi A e_{WL}} \tag{4}$$

$$e_{WL} = \frac{A_{eff}}{A} \cdot e = \left(\frac{b_{eff}}{b}\right)^2 \cdot e$$

Hence

$$e_{WL} = \left(1 + 2\frac{h}{b}\right)^2 \cdot e \quad . \tag{5}$$

This simple geometrical consideration aids in understanding the phenomenon, but it is not accurate enough, because it would yield the same result for any tip device with ratio h/b. Proposed is a penalization via a factor k_{WL}. The height of the winglet is divided by a certain parameter. This is exactly the parameter or "**ratio**" that appears again and again in literature and which was discussed in Chapter 1.4 (where it appeared as 1.5 or 2.0).

$$e_{WL} = \left(1 + \frac{2}{k_{WL}}\frac{h}{b}\right)^2 \cdot e = k_{e,WL} \cdot e \quad , \quad k_{e,WL} = \left(1 + \frac{2}{k_{WL}}\frac{h}{b}\right)^2 \tag{6}$$

If the winglet with its height has the same effect as a span increase, then k_{WL}=1.0 and Eq. (6) is the same as Eq. (5). This is the geometrical equivalence of the winglet. I.e. the winglet sticking up is as good as folding it down. If, however, the winglet height needs to be divided by 2 and only this reduced height taken as a span increase gives the performance of the winglet, then k_{WL}=2.0. This is what is proposed by McLean [7] and Howe [32]. The inverse $1/k_{WL}$ is called the "**intrinsic aerodynamic efficiency of the winglet**". It is independent of its height and shows simply how good the winglet was designed. e is the span efficiency of the basic wing, e_{WL} is the span efficiency of the wing with winglet. $k_{e,WL}$ is the winglets contribution to span efficiency of the wing with winglet.

STEP 3: $$k_{WL} = 2\frac{h}{b} \cdot \frac{1}{\sqrt{k_{e,WL,v}} - 1} \tag{7}$$

3.2 Preparing the Efficiency Calculation

It can well be that the winglet (as in Fig. 4) is not straight up, but has an outward cant angle. In this case it is necessary to eliminate the horizontal effect with Eq. (8). It remains only the vertical winglet contribution to span efficiency

STEP 2: $$k_{e,WL,v} = \frac{k_{e,WL,total}}{\left(1+2\frac{h_h}{b}\right)^2} \quad (8)$$

Usually, the relative total drag reduction

$k_{D,WL} = \Delta C_{D,WL}/C_D$ is given (in %, e.g. for the Sharklet: 4%, Appendix or Table 2). (9)

It consists of a larger negative difference (reduction) due to induced drag $\Delta C_{Di,WL}$ and a smaller positive difference (contribution) due to zero-lift drag, $\Delta C_{D0,WL}$. Both together are the total change in drag due to winglets $\Delta C_{D,WL} = \Delta C_{D0,WL} + \Delta C_{Di,WL}$ which should be negative.

$k_{Di} = C_{Di}/C_D \approx 0.4$ [33] and $k_{D0,WL} = \Delta C_{D0,WL}/C_{D0} \approx 0.038$ [13] . (10)

If we make the simple assumption (often the only possible due to lack of data) that $\Delta C_{D,WL} = \Delta C_{Di,WL}$:

STEP 1: $$k_{e,WL,total} = \frac{1}{1+\frac{k_{D,WL}}{k_{Di}}} \quad (11)$$

Alternatively, if we make the assumption (with more data available) that $\Delta C_{D,WL} = \Delta C_{D0,WL} + \Delta C_{Di,WL}$:

STEP 1: $$k_{e,WL,total} = \frac{1}{1-\left(\frac{1}{k_{Di}}-1\right)\cdot k_{D0,WL}+\frac{k_{D,WL}}{k_{Di}}} \quad (12)$$

With the assumption $\Delta C_{D0,WL} = 0$ and hence $k_{D0,WL} = 0$, Eq. 12 simplifies to Eq. 11.

$$k_{Di} = 1-\frac{1}{1+\frac{1}{(V/V_{md})^4}}$$, from aircraft design: $1 \le V/V_{md} \le \sqrt[4]{3} = 1.316$ (13)

$k_{Di} = 0.4$ as proposed by Kroo [33]. It can be shown that this is true for $V/V_{md} = 1.11$, certainly a typical value in aircraft design. All derivations and background information is given in [13].

The method has been applied to **Whitcomb's winglet** from Fig. 4. A $\boldsymbol{k_{WL} = 2.7}$ was obtained [13]. This is well in the usual range. The intrinsic aerodynamic efficiency is 1/2.7 = 0.37 only. As such it is not aerodynamically superior to a span increase! This is in contrast to what Whitcomb seems to convey in [18].

3.3 Intrinsic Efficiency of Winglets Calculated from Data in Literature

Thanks to data especially from Boeing [14], it is possible now to calculate k_{WL} and the Intrinsic Efficiency of Winglets. We use STEP 1 and go with $k_{e,W,total}$ directly into STEP 3, because we assume that there is no horizontal extension, or it is already included into is 2 *h/b* as given in Table 2.

Table 2: k_{WL} calculated for selected passenger aircraft. Visualization in Fig. 6.

aircraft	2 *h/b*	relative drag reduction in cruise	k_{WL}
767-400 raked tip	10.5%	5.5%	1.4
747-400 tip plus winglet	12.5%	3.5%	2.7
A320neo Sharklet	14.1%	4.0%	2.6
737-800 blended winglet	14.3%	3.8%	2.8
KC-135 winglet	14.5%	4.5%	2.4
MD-11 extended winglet	15.5%	3.5%	3.3

In Fig. 6, the pure geometric consideration yields a curve for $k_{WL} = 1$. Howe [32] and McLean [7] assume that $k_{WL} = 2$ as a rule of thumb. Dubs [12] and Zimmer [33] consider induced drag only and assume that there is no zero-lift drag. Real aircraft (A/C) are less efficient, because they experience also an unknown zero-lift drag increase; this is embedded into k_{WL}: It is assumed that the whole drag reduction phenomenon can be described by a reduction in induced drag alone. Due to additional zero-lift drag the induced drag reduction appears less strong and hence k_{WL} appears a little larger compared to a calculation where the zero-lift drag increase is considered separately (as in the alternative STEP 1).

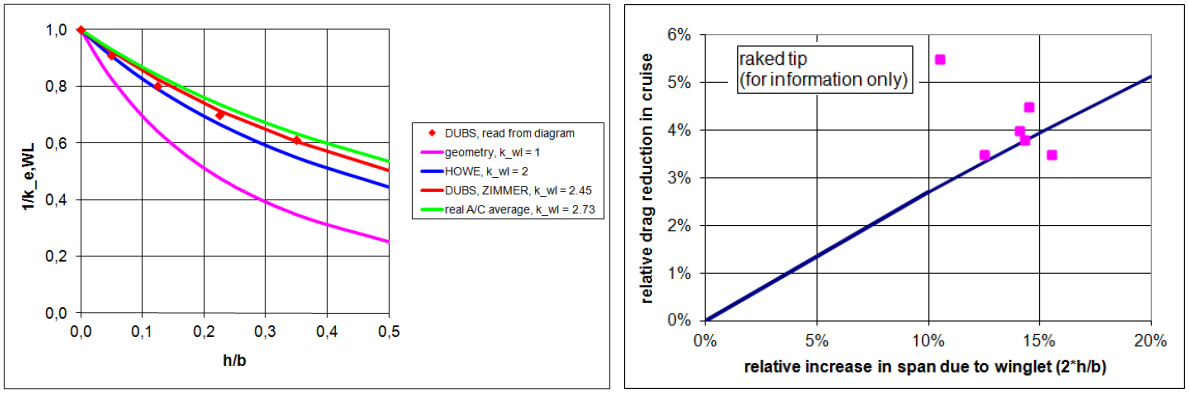

Figure 6: $b/b_{eff} = C_{Di}/C_{Di,ref} = 1/k_{e,WL}$ as a function of h/b from different authors [12][32] [33]. Total relative drag reduction in cruise as a function of increased span due to winglets ($2 \cdot h$) with data from [14], [34]. Parameters in Table 2. See also [31] for detail.

4 ESTIMATING MASS INCREASE DUE TO WINGLETS

With this Chapter, we go beyond the title of the paper. The mass increase due to winglets, Δm is estimated from "beefing up" the wing, resulting in Δm_{beef} and the additional mass due to the two winglets (left and right wing tip) themselves, Δm_{WL}.

$$\Delta m = \Delta m_{beef} + \Delta m_{WL} \tag{14}$$

$$k_{m,beef} = \frac{\Delta m_{beef}}{m_{CR}}\ ,\ m_{CR} = \frac{m_{MTO} + m_{MZF}}{2}\ ,\ \frac{k_{m,beef}}{k_{D,WL}} = 0.1 \ldots 0.5 \quad \text{([7]: B737NG ... Fig.4.3)}$$

$$\Delta m_{beef} = k_{m,beef} \cdot m_{CR}$$

Variation 2 (derived from [22] with details of the derivation in [13]):

$$\Delta m_{beef} = 0.44(k_{e,WL,total} - 1) \cdot m_{W,ref}$$

Variation 3 (with $m_W(x)$ being any kind of equation or method to estimate the wing mass):

$$\Delta m_{beef} = m_W(b_{eff}) - m_W(b)$$

$$\frac{\Delta m_{WL}}{h} = 83\ \text{kg/m} \ \ldots 111\ \text{kg/m} \qquad \text{(A320neo [31] ... B737NG [7])}$$

$$\frac{\Delta m_{WL}}{S_{WL}} = 180\ \text{kg/m}^2 \ldots 200\ \text{kg/m}^2 \qquad \text{(A320neo [31] ... B737NG [7])}$$

$$h = \frac{b}{2} k_{WL}\left(\sqrt{k_{e,WL,v}} - 1\right) \qquad S_{WL} \approx h \frac{c_t}{2}$$

c_t: cord of wing tip;

assumed chord at tip of winglet: $c_{WL,t} \approx 0$

$$\Delta m_{WL} = \frac{\Delta m_{WL}}{h} \cdot h \qquad \text{or} \qquad \Delta m_{WL} = \frac{\Delta m_{WL}}{S_{WL}} \cdot S_{WL}$$

5 ESTIMATING THE DRAG AND FUEL BURN REDUCTION DUE TO WINGLETS

The speed polar $D = f(V)$ is used when dealing with aircraft performance. The speed polar follows directly from the lift and drag equation and contains the terms (just abbreviations) called A and B [26]. The speed polar is

$$D = AV^2 + BV^{-2} \quad \text{where} \quad A = \frac{1}{2}\rho\, C_{D0}\, S_W \qquad B = \frac{2m^2 g^2}{\rho\, S\, \pi\, A\, e} \tag{15}$$

$$D_{WL} = A_{WL}V^2 + B_{WL}V^{-2} \qquad A_{WL} = \frac{1}{2}\rho\, C_{D0,WL}\, S_W \qquad B_{WL} = \frac{2(m+\Delta m)^2 g^2}{\rho\, S_W\, \pi\, A\, e\, k_{e,WL}}$$

Drag reduction: $\Delta D = D - D_{WL}$ $\qquad A_{WL} = A$, if assumed that $C_{D0,WL} = C_{D0}$.

Relative fuel burn reduction $\dfrac{\Delta m_F}{m_F} = \dfrac{\Delta D}{D}$

follows from drag reduction including also the effect of mass increase and zero-lift drag increase.

6 SUMMARY

A method has been presented to calculate the "intrinsic aerodynamic efficiency of winglets" $1/k_{WL}$. It lumps all aerodynamic winglet characteristics (from zero-lift drag and from induced drag) into this single parameter. A constant typical cruise lift coefficient is assumed because changes in aircraft mass are not considered. If e.g. the winglet needs to be 3 times larger compared to a horizontal span increase with the same overall aerodynamic effect ($k_{WL} = 3$), its intrinsic aerodynamic efficiency would be the inverse or 1/3. A simple method is given to estimates the mass increase due to winglets. Finally, the overall drag reduction can be calculated from the speed polar. This yields also an estimate of the relative fuel burn reduction due to winglets.

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APPENDIX

Case Study: Airbus A320neo Public Performance Statements

Airbus has added so called "Sharklets" as retrofit on the A320ceo or installed by default on the A320neo. The information is communicated only in press releases. It seems not to be communicated in aircraft specifications or scientific literature. Airbus' press releases reveal: "Operators of Sharklet retrofitted aircraft will benefit from a reduction in fuel costs by up to 4%" [35]. "The delivery ... for American Airlines, ... is the very first A319 to feature Sharklets ... that offer up to 4% fuel burn savings" [36]. Does 4% make sense? Let's check further: "The A320neo is a new engine option for the A320 Family ... and incorporates latest generation engines and large 'Sharklet' wing tip devices, which *together* will deliver 15% in fuel savings [with respect to the A320]" [37] [38]. The A320neo is offered with two engine options: Pratt & Whitney PW1000G and CFM International LEAP-1A. About

the Pratt & Whitney PW1000G it is known: "The PurePower PW1000G engine family improves fuel burn up to 16% versus today's best engines" [39] and "fuel-burn performance is ... 16% better than the International Aero Engines V2500 baseline [as used on the A320]" [40]. About the CFM International LEAP-1A it is known: "The LEAP-1A ... for the next-generation single-aisle airliner from Airbus, the A320neo. It offers A320 operators ... a 15% reduction in fuel consumption" [41]. "This advanced new turbofan will reduce the engine contribution to aircraft fuel burn by up to 16% compared to current CFM56 Tech Insertion engines that power Airbus A320" [42]. The engine manufacturers most probably refer to the engine's specific fuel consumption (SFC). When we compare the engines on aircraft, we have to consider their *difference* in drag due to engine installation. Estimated (based on [43]), this is certainly less than 1%. So we must conclude: The A320neo is 15% better in fuel burn than the A320ceo. This performance improvement is due to the new (installed) engines alone. Hence, based on Airbus' information, the winglets on the A320neo seem not to have an effect in combination with the new engines, but this is a contradiction to the (somewhat) plausible number of 4% drag reduction (see Fig. 6)! A case study related to the winglets and the new engine of the Airbus A320neo is included in [44]. The A320neo burns on its DOC mission 14.4% less fuel when payload is kept constant. Direct Operating Costs (DOC) are reduced by about 2.5%.

General Criticism of the Industry's Public Aircraft Performance Statements

All this should just substantiate that reporting of aircraft performance as apparently done today is far from satisfying. Messages are placed in the media often by salesmen with the intent to shape the image of their aircraft as a product in public. No other information is openly available. "Fuel burn" and "fuel consumption" for an engine manufacturer is different from the same terms used by an aircraft manufacturer. For the latter, "fuel burn" is mostly meant over a certain range (which is usually not stated), but could also be meant as an instantaneous fuel burn calculated from Specific Air Range as 1/SAR (usually without specifying at what aircraft gross mass it is given). Winglets are praised even if they may take their potential mostly from a span increase.